
Secondary Mathematics Teachers' Understanding of the Nature of Mathematics

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Abstract

Pre-service secondary mathematics teachers should develop rich understandings of the nature of mathematics through their completion of undergraduate teacher preparation programs. But what specifically should we be teaching pre-service secondary teachers about the nature of mathematics? And how can we ensure pre-service secondary teachers effectively learn about the nature of mathematics? Researchers in science education are conducting systematic inquiries into teachers' understandings of the nature of science, but comparatively less research on the analogous topic has been conducted in mathematics education. This paper presents findings from an exploratory study, the primary goal of which was to create a humanistic framework for the nature of mathematics that outlines potential goals for students' understanding of the nature of mathematics.

Introduction

The Mathematics Teacher Education Partnership's Guiding Principles for secondary mathematics teacher preparation programs state:

Nature of mathematics: The secondary mathematics teacher preparation program ensures teacher candidates understand, and are able to convey to their students that mathematics is a living and evolving human endeavor that relies on logic and creativity, and it is valuable for citizenship, for the workplace, as well as for its intrinsic interest.

This indicator of Guiding Principle #4, Candidates' Knowledge and Use of Mathematics, emphasizes the importance for secondary mathematics teacher preparation programs to provide future teachers opportunities to learn about the nature of mathematics in ways that will subsequently influence their teaching practice. These opportunities aim to impact secondary students' perceptions of mathematics in productive ways; however, the field of mathematics teacher education still has much work to be done in this area.

Students and teachers may have limited views of the nature of mathematics; they may view mathematical knowledge as static and consider the primary practice of mathematics to be the memorization of rules and procedures (Beswick, 2012; Erlwanger, 1974; Muis, 2004; Presmeg, 2007; Thompson, 1992). These naïve views may negatively affect the teaching (Thompson, 1992; White-Fredette, 2010) and learning (Erlwanger, 1974) of mathematics. These views are referred to as naïve because they are in stark contrast to those held by mathematicians and mathematics education scholars. For example, Boaler (2016) noted,

Mathematics is a cultural phenomenon; a set of ideas, connections, and relationships that we can use to make sense of the world. At its core, mathematics is about patterns. We can lay a mathematical lens upon the world, and when we do, we see patterns everywhere; and it is through our understanding of the patterns, developed through mathematical study, that new and powerful knowledge is created. (p. 23)

Although I am not aware of any studies that specifically describe future secondary teachers' understandings of the nature of mathematics, those of us who teach this group know that many students pursue a mathematics major because they view mathematics as a subject of right and wrong answers for which one must

develop the skill of memorizing and applying correct procedures. Given what we know about future secondary mathematics teachers' prior learning experiences, in which much of their schooling may have included traditional instruction in preparation for standardized tests, many have likely not experienced the creative aspect of mathematics (Burton, 1999).

Coming back to the MTE-Partnership goals, we need to consider this question: What instructional practices are effective to teach future secondary teachers that “mathematics is a living and evolving human endeavor that relies on logic and creativity, and it is valuable for citizenship, for the workplace, as well as for its intrinsic interest”? (MTE-Partnership, 2014, p. 3). What specifically do students need to learn if they are to see mathematics as an evolving human endeavor? We cannot meet this broad goal if we do not express it in more detail. Our field needs a systematic inquiry into the nature of mathematics.

The Nature of Science

If we, as scholars in mathematics education, are to conduct a systematic inquiry into the teaching and learning of the nature of mathematics, we would do well to learn from the field of science education. Within science education, scholars have long recognized that students have many misconceptions about the nature of scientific knowledge and the practice of scientists (Hurd, 1960). For instance, colloquial use of the word *theory* (as a whimsical guess rather than an explanation with substantial confirming evidence), contributes to students' stance that evolution is *just a theory*. To address such misconceptions, and promote a humanistic view of the scientific enterprise, scholars have created lists of goals for students' understandings of the nature of science (Lederman & Lederman, 2014). For instance, the Next Generation Science Standards (NGSS, 2013) contain an appendix that lists, in detail, understandings of the nature of science that K–12 students should achieve. Students should understand, for example, that “Scientific knowledge is open to revision in light of new evidence” (p. 4). The NGSS provides detailed descriptions of how student understandings of these aspects of science should unfold across all K–12 grade levels. In addition to lists that outline goals for students' understanding of the nature of science, scholars are also conducting research to understand students and teachers' conceptions of the nature of science (Lederman, Abd-El-Khalick, Bell, & Schwartz, 2002). Other studies examine the instructional practices that are effective in teaching specific nature of science characteristics (Lederman & Lederman, 2014). The purpose of this study was to begin the conceptualization of the nature of mathematics as has been done with the nature of science in science education, hopefully leading to a systematic inquiry into the subject.

Methodology

This paper reports on a dissertation study in which I sought to begin a systematic inquiry into the nature of mathematics—analogous to that which is ongoing in science education—and produce the beginnings of a humanistic framework for the nature of mathematics that would be revised in the future. Two main questions guided the study: (1) What is the nature of mathematics? and (2) What should undergraduate students in a transition-to-proof course—including future secondary mathematics teachers—understand about the nature of mathematics?

Before the study was conducted, I conducted a thorough literature review into the nature of mathematics within mathematic education research and selected pieces from humanistic philosophy of mathematics (e.g. Hersh, 1997). For the study, I conducted a heuristic inquiry (Moustakas, 1990), collecting and analyzing data from a variety of sources in order to form a creative synthesis that captures the human essence of the nature of mathematics as it emerged during the study. Describing heuristic inquiry, Patton (2015) noted:

[H]euristic research epitomizes the phenomenological emphasis on meanings and knowing through personal experience; it exemplifies and places at the fore the way in which the researcher is the primary

instrument in qualitative inquiry; and it challenges traditional scientific concerns about researcher objectivity and detachment. (p. 119)

I chose this methodology because I believe that to know mathematics is to do mathematics. In order to personally understand the nature of mathematics, I considered my experience of mathematics in all aspects of my life. I also pursued experiences that would be highly relevant to the nature of mathematics. I studied my own experience doing mathematics through collaboration with a professional mathematician working to prove an unsolved conjecture in graph theory. I documented this collaboration through audio recordings of discussions with the mathematician and hard-copies of all mathematical work. Throughout this work, I considered my two research questions, identifying the meaningful things I was learning about the nature of mathematics and considering what aspects would be valuable for students to understand about the nature of mathematics.

Another key component of data came from an undergraduate transition-to-proof course. Transition-to-proof courses are an important part of many universities' preparation programs for future secondary teachers majoring in mathematics education. In the course that was involved in this study, six of the 23 students were mathematics education majors. The course was co-taught by two mathematics education scholars. The first, an assistant professor of mathematics education, designed and taught the course for seven prior semesters. The second, the author of this paper, was a doctoral student completing this study as a dissertation project. Both instructors frequently utilized their understandings of the discipline of mathematics to help students understand the nature of pure mathematics and proof.

I also collected several other sources of data. I conducted several interviews with mathematicians and others, the general topic of these interviews being the nature of mathematics and students' understandings of the nature of mathematics. I also kept a detailed reflective journal and personal audio recordings, which served as a means to analyze my experience of the nature of mathematics as it unfolded in all aspects of life. The data was analyzed using several qualitative methods, including initial coding and heuristic inquiry methods such as immersion, illumination, explication, and creative synthesis. The product of the study is a list of humanistic features of the nature of mathematics that can be researched as appropriate goals for students' understanding of the nature of mathematics along with real-life narratives, which put these humanistic features into context.

Data Analysis

The key goals of the study were to form a list of characteristics of the nature of mathematics that may serve as goals for students' understanding of the nature of mathematics and to form a creative synthesis (Moustakas, 1990) that consisted of real-life stories grounded in the data that illustrated each of these characteristics, putting a human face on mathematics. In this spirit of heuristic inquiry (Moustakas, 1990; Patton, 2015), I analyzed data both during and after data collection. During data collection, I constantly reflected on what I was learning about the nature of mathematics through data collection and considered what would be valuable for students to understand about the nature of mathematics. Often, I would take the opportunity to discuss my ideas with others during the informal interviews with mathematicians and others, getting opinions about what is valuable for students to understand about the nature of mathematics. By the end of data collection, I had created a large list consisting of possible goals for students' understanding of the nature of mathematics.

After data was collected, my main goal was to finalize a list of characteristics of the nature of mathematics that would be valuable for students to understand, and tell real-life stories using excerpts from my data that illustrated each of the characteristics in the list/framework. To begin this process, I transcribed all of the collected data. I then assigned a code to each possible goal for students' understanding of the nature of mathematics from the list created during data collection. I then used the qualitative data analysis software ATLAS.Ti to code the

entire data set, noting when certain characteristics were reflected in the data. I then wrote stories that illustrated each of the characteristics that were emerging as most meaningful in the study. Lastly, I finalized a framework together with several illustrative narratives.

Results

The list in Table 1 is a major product of the study, which outlines characteristics of the nature of mathematics that may be valuable for undergraduate students—including future secondary teachers—to know and understand. The characteristics and categories should be studied in future research. The list is divided into three sections: Mathematics as a fundamental part of human cultures; the IDEA framework for the nature of pure mathematics; and statistical and applied mathematics. I chose not to include the illustrative stories in this manuscript due to space limitations, but the interested reader can find them in my dissertation (Pair, 2017). The framework and some of its corresponding characteristics will be discussed in the remainder of the paper. In that discussion, the list in Table 1 will be referred to as the NOM Framework, NOM being an acronym for the nature of mathematics.

Table 1

NOM Framework

Mathematics as a Fundamental Part of Human Cultures
<ul style="list-style-type: none"> • Western academic mathematics is one (but not the only) form of mathematics • Mathematical knowledge is influenced by cultural values. • Mathematical knowledge is embedded within the work of artisans. • The purpose of commercial-administrative mathematical knowledge is calculation for economic purposes; the efficiency and accuracy of mathematical procedures is valued.
The IDEA Framework for the Nature of Pure Mathematics
<ul style="list-style-type: none"> • Our mathematical ideas and practices are part of our <i>identity</i>. • Mathematical knowledge is <i>dynamic</i> and forever changing. • Pure mathematical inquiry is an <i>exploration</i> of ideas. • Mathematical ideas and knowledge are socially vetted through <i>argumentation</i>. <p style="text-align: center;"><i>Secondary Characteristics</i></p> <ul style="list-style-type: none"> • Proofs are bearers of mathematical knowledge. • Mathematics can be emotional and creative. • Informal mathematical work is foundational to formal knowledge. • Mathematicians change focus in a mathematical situation to achieve insight.
Statistical and Applied Mathematics
<ul style="list-style-type: none"> • Mathematical knowledge is used to shape society, but cannot be considered an absolute judge.

Discussion

The ensuing discussion connects the NOM Framework in Table 1 to both the MTE-Partnership (2014) principle highlighted at the beginning of this paper, as well as to Rochelle Gutiérrez's descriptions of humanizing mathematics (as suggested by a reviewer of this paper). Implications for secondary teacher education programs will also be incorporated. Note that the NOM Framework does not outline students' conceptions of the nature of mathematics, but rather forms a list of possible goals for students' understandings of the nature of mathematics that can guide instruction and research.

Mathematics is a Living and Evolving Human Endeavor

I set out to create NOM framework that was grounded in humanistic philosophy of mathematics. I wished to counteract the testing culture and the trend of viewing mathematics as an abstract value-free static body of knowledge. The MTE-Partnership (2014) indicator states that teacher candidates “are able to convey to their students that mathematics is a living and evolving human endeavor that relies on logic and creativity” (p. 3). Regarding the NOM Framework, we can see that mathematics is a living and evolving human endeavor in several areas. The first major category, mathematics as a fundamental part of human cultures, highlights the human aspect of mathematics in everyday life. The second category describes the discipline of pure mathematics; the evolving nature of mathematical knowledge is captured within the notion that mathematical knowledge is *dynamic* and forever changing. Gutiérrez (2012) also highlighted the importance for teachers to see mathematics as dynamic:

Unfortunately, many teachers are not aware of the uncertainty that is present in mathematics. Ask any person on the street to describe the nature of mathematics and you will hear words like “black and white,” “absolute,” “one right answer,” “truth,” leaving you with the idea that mathematics is static and predetermined. Yet, talk to a mathematician and you will learn that mathematics is constantly changing and does not always give one right answer. (p. 38)

Regarding mathematics teacher education, we need to consider ways to structure our programs so that future teachers begin to understand the dynamic nature of mathematical knowledge. Future secondary teachers should understand that new mathematical knowledge is being created every day and understand that their knowledge will be evolving throughout their lives.

Mathematics is Valuable for Its Intrinsic Interest

The MTE-Partnership (2014) document states that teachers need to help their students understand how mathematics can be pursued “for its intrinsic interest” (p. 3). During my own experience conducting mathematical research in this study, I found mathematics to be a very enjoyable exploration of ideas. After learning about this research, some of the students that participated in the study reacted with interest. But one pre-service secondary teacher said, “Why are you doing this? What is the point? I just want to teach high school algebra.” This student was not interested pursuing mathematics for its own sake. She believed that pursuing work on an unsolved mathematics problem was not relevant to her future career as a teacher. How can our secondary teachers inspire a love of mathematics learning in their students if they do not find value in pursuing mathematical ideas themselves?

The concept of joy while doing mathematics is central to Gutiérrez’s (2017a) vision of mathematics, which she calls *mathematx*.

Mathematx is a way of seeking, acknowledging, and creating patterns for the purpose of solving problems (e.g., survival) and experiencing joy. Beginning with the principles of recognizing self and/in others, responsibility towards others, and valuing tensions, several things stand out as different from the typical way Western mathematics is conducted or experienced by students in school. First, although some mathematicians experience pleasure as a result of solving previously unsolved problems, that aspect of joy is often a very small percentage of the time and almost always absent from the “mathematical product” (e.g., new theorem, new proof) that is valued by the community. Yet, mathematics education researchers who study aesthetics highlight this domain as essential to human meaning making and to the insights that mathematicians develop. (p. 15)

In addition to an emphasis on joy, the previous quote also highlights the personal and interdependent nature of mathematics. This again connects to the NOM framework and the notion that our mathematical ideas are part of our *identity*.

While I have pointed out some similarities between my work and Gutiérrez's work in humanizing mathematics, there are also some contrasts. Whereas my motivation was to develop a vision of mathematics that might restructure mathematics classrooms so that mathematics is not something we must suffer through, but something we can enjoy, Gutiérrez's conception challenges us to consider a world in which we change our fundamental human interactions to decolonize. As a white male educator, I may not always be aware of the way I am pushing the status quo. Gutiérrez recently argued that "mathematics operates as whiteness when we do not acknowledge the contributions of all cultures" (2017b, p. 8). The NOM framework that I have created may privilege Western pure mathematics too much, and not highlight the mathematics of other cultures (or living beings as Gutiérrez puts it) enough. My NOM framework does highlight mathematics as a fundamental aspect of human culture, and that Western mathematics is only one form of mathematics, yet a significant piece of the framework describes pure mathematics. Regarding pure mathematics, Gutiérrez (2017a) noted

When we use terms like pure mathematics or fundamental mathematics, we are 'othering' different forms of mathematics in ways that make them sound primitive or deviant. An Aboriginal stance would call into question whether any form of mathematics could be seen as pure, as it will always have a purpose and a grounding—cultural context—to start. (p. 16)

Gutiérrez's goal is to fight against colonization and find a way to do mathematics in a way that values interdependence and living beings learning from one another. She (2017b) writes,

I mean a program that takes seriously land, sovereignty, and the history of erasure of people through culture and language. I acknowledge the ways in which mathematics teaching and learning contributes to the denial of language and history for Indigenous students primarily. First, we must begin by acknowledging settler colonialism and ask whose history and whose language is part of mathematics? We cannot claim as our goal to decolonize mathematics for students who are Black, Latinx, and Aboriginal while also seeking to measure their "achievement" with the very tools that colonized them in the first place. (p. 12).

Gutiérrez challenges us to unlearn what we think we know about the nature of mathematics and consider alternative definitions of mathematics. For Gutiérrez, mathematics is the science of patterns, but the patterns she speaks of extend beyond those typically studied in a mathematics class (Gutiérrez, 2016). She suggests that we recognize that even trees notice patterns in their environment and are able to communicate in response to those patterns with new patterns, thereby benefitting neighbor trees of their own and different species (2017a). Indeed, she claims the plants and rocks are our ancestors, and we may be able to learn from them as they can learn from us. Or that we can learn from mathematics and mathematics can learn from humans. Then even further, we can imagine new patterns of interaction within our world and conceive of what actions would need to take place to bring about those patterns. In spite of these apparent differences, I believe both Gutiérrez and I have similar goals regarding peace for life on earth, and I will continue to reflect on how my work on the nature of mathematics connects with hers.

Conclusion

To say that students have naïve understandings of the nature of mathematics may be misguided. Their understandings of the nature of mathematics are a consequence of their experiences with mathematics. The nature of the mathematics that school students have experienced has a particular nature, and students may have accurate conceptions, descriptions, and beliefs about it. But with this study, I have aimed to discuss a different perspective of mathematics. In terms of teacher education programs, we must structure our learning environments so that future teachers come to see an alternative view. In order for school students to experience and view mathematics differently, it is a necessary but not sufficient condition that their teachers have had the opportunity to view mathematics differently. Teacher educators can play a role in bringing change about, by presenting future teachers with experiences that challenge their prior conceptions of mathematics and show them that a different mathematics is possible.

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