
Active Learning in a Number Sense Course for Future Mathematics Teachers

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Abstract

We report how the recommendations from the Conference Board of the Mathematical Sciences' *The Mathematical Education of Teachers* (2001) and *The Mathematical Education of Teachers II* (2012) have been implemented through a unique course. This Numbers and Operations-focused course, "Number Sense for Middle and High School Teachers," addresses among other things the concern that teachers be able to assist students in explaining the "why" behind the mathematics typically taught at the K–8 level. Furthermore, we discuss how we believe that using active learning strategies has impacted the future teachers who are students in this course. Finally, we discuss the impact on recruitment of students in a Noyce Track 1 Scholarship program. We believe that the course's focus on developing a deep understanding of topics provides the framework for future teachers to more fully understand many of the topics and underlying principles of mathematics taught at the secondary level as well. The active learning format of the course provides students an environment in which they can feel comfortable exploring subtly challenging mathematics and models good pedagogy for these future teachers. Students learn the mathematical practices that are related to problem solving, such as embracing failure and struggle, and valuing the process of discovery. Exposure to the type of sense-making content and active learning pedagogy in the course also seems to encourage students to be willing to be more involved in a Noyce Track 1 Scholarship program.

Keywords: Teacher preparation, number sense, conceptual understanding, active learning

Introduction

The University of Nebraska at Omaha (UNO) has collaborated with other universities engaged in the Mathematics Teacher Education Partnership (MTE-Partnership) to create institutional change in the way teachers learn mathematics at UNO. The Research Action Cluster (RAC) within the MTE-Partnership focused efforts on implementing active learning in the Precalculus through Calculus 2 (P2C2) courses. These efforts have influenced the adoption of such strategies in new courses at UNO, including a new course developed for middle school and secondary math teachers. The focus was on developing deep understandings of the topics frequently taught at the K–8 level with active engagement of students in a rich mathematics class. The course would provide students a new understanding of mathematical problem solving, use of definitions, and strategies that would result in strengthened preparation to be future mathematics teachers. The two main goals of the implementation were (1) to create deep understanding of many topics taught at the K–8 level along the connections to topics encountered in high school, and (2) to serve as an instructional model for future mathematics teachers in active learning. Finally, we will discuss a third benefit that is emerging from the implementation of active learning courses in the new course and others, that is, improved recruiting efforts into Noyce Track 1 Scholarship programs.

Mathematics Coursework for Future Teachers

To begin, an explanation of the first goal discussed previously will be provided. To do this properly, a detailed examination of the course rationale, development, and content will be provided, as the course itself is unusual. *The Mathematical Education of Teachers II* (MET II; 2012) recommended six semester hours in Numbers and Operation for future middle school teachers. Since many of states have primary endorsements to teach mathematics that is a combined middle/high school level certificate, the report implies that in these states all future teachers of mathematics should take these six semester hours. However, in 2014, Newton et al. reported that among 64 teacher preparation programs responding to a survey regarding course requirements, virtually none of them had a course on Numbers and Operations on the K–8 level. From this data, it is clear that the MET II recommendations have not been enacted on a comprehensive level. There are many other reasons why a 6–12 teacher would benefit from exposure to such content. Since many middle school students and high school students struggle with material and concepts developed from lower grades, their teachers frequently teach courses that are cover topics from earlier years. Moreover, future 6–12 teachers can benefit from a stronger foundation in Numbers and Operations because of the carryover effect in their understanding of related topics in algebra and more traditional high school content.

The UNO Course Content

The Number Sense course developed at UNO is an exception to the trend nationally to not include a course on numbers and operation. The course is required for the secondary and middle school math education endorsements. The course uses the textbook *Mathematics for Elementary Teachers with Activities* by Sybilla Beckmann, which despite the title, covers K–8 content (Beckmann, 2014). For this course, topics that were identified as seventh- and eighth-grade topics and not normally covered in the elementary version of the course, such as proportional reasoning, were included. Topics that are firmly in the K–6 curriculum but are known to be those that students struggle with in future years, such as fractions, were included. Students in the class tackle the tasks of “why” certain algorithms work the way they do, how to represent different mathematical ideas, and various ways to justify their conclusions. Students also encounter problems solved with multiple strategies, as opposed to believing there is only one way to solve a given problem. They develop mathematical habits of mind that are a basis to good problem-solving skills (Sword et al., 2018).

Modeling Effective Teaching Practices

A second goal of this course and this paper is related to the course instruction serving as an instructional model for future mathematics teachers in active learning. The integration of pedagogy and content in courses designed for future mathematics teachers has been identified as an important part of a quality teacher preparation program as identified by the National Mathematics Advisory Panel (2008) and the National Research Council (2010). Dating back to 2003, Ferrini-Mundy and Graham noted that over the years questions have been raised, not just about the nature and extent of the mathematics courses required by teacher education programs, but also about the integration of mathematics and pedagogy.

Attention to pedagogy used in mathematics content courses is imperative. It is often said that “teachers teach as they were taught” (Hall et al., 2006). Current research reveals that teachers actually teach in the way they preferred to be taught, or the way they believe their students will learn best (Cox, 2014). Teachers are more likely to integrate high-quality mathematical practices of teaching (Cobb et al., 2011), when they have productive experiences modeling such practices first as students in a college classroom. The integration of active learning principles in a Number Sense course will expose students to this form of instruction.

Extending Active Learning into Content Courses for Teachers

The use of active learning at UNO can best be described by the concept of inquiry-based learning (IBL). IBL, according to the Academy of Inquiry Based Learning (2018), is a learner-centered method that deeply engages

students and in which students collaborate on their learning in some fashion. In the UNO course, students do more than take notes and write down definitions. Rather, they tackle tasks that require them to engage with one another, experiment, and communicate their mathematical ideas to one another. Students are encouraged to embrace the attitude in which failure and struggle is not stigmatized, but rather, the process of discovery is encouraged, and thus they learn to value perseverance more and develop a growth mindset. The role of the instructor is to guide the students in their sense-making. IBL promotes the mathematical practices called for by a variety of organizations and in the Common Core standards (see Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics (NCTM), 2012a; NCTM, 2012b).

The UNO Class Educational Philosophy and use of IBL

Several years ago, the authors went through significant training in the use of IBL in the classroom. As part of the RAC initiatives, some Calculus and Precalculus classes – required classes for future teachers – have integrated IBL into the instruction at UNO during the past five years. The impact of IBL instruction on students has been mostly positive, and the decision was made to utilize active learning in other required courses for future teachers. IBL was utilized in teaching a History of Mathematics class, which was required of all secondary math majors (Matthews & Hodge, 2016). An active learning class in number sense appeared to be a sensible next step in providing a good preparation for mathematics teachers and so IBL was intentionally implemented into the course structure.

In the class, students are inundated with opportunities to develop mathematical habits of mind and to communicate mathematical reasoning. The active learning format of the course provides students an environment in which they can feel comfortable exploring topics in K–8 mathematics from an advanced perspective that pushes even the strongest students to pay attention to precision, definitions, and reasoning. The ultimate goal was a well-crafted solution through an adventure in mathematical discovery (Kogan & Laursen, 2014).

The use of IBL and active learning in the Number Sense course helps model the mathematical practices called for in various national standards, providing students an opportunity to be inspired to become teachers that utilize active learning in their own classrooms. The premise is that students who are thus inspired are more likely to engage their students in active learning in future classrooms.

The Classroom Environment/Typical Day Structure

Changes to the physical environment facilitated active learning. Traditional desks were replaced by tables and chairs. The arrangement encouraged students to focus their attention on one another and on their personal exchanges, rather than on “the sage on the stage.” Students were initially in self-selected groups, but some instructors switched groups throughout the semester.

As class begins, table groups determine if any significant questions remain from the select homework problems. These homework problems were chosen because they had important concepts embedded within the problem. After the class agreed on two or three problems to be discussed as a group, students volunteered to present their solutions to the class. Often a student would say, “I’m not sure if I have it right, but I’ll put up what I have.” This example is a great testimony to the strong support students give to one another and their willingness to work cooperatively. Often, more is learned from the presentations with errors than those that are correct. It was frequently frustrating to students as they struggled with topics that they considered quite basic, such as fraction operations and problems related to properties of arithmetic. As the students presented their work and provided detailed explanations, the teacher of the class felt that students developed deeper understandings of the topics. Most students believed they “knew how to do the math,” but learned they didn’t necessarily understand the “why” behind the strategies and procedures used in K-8 topics before the class.

After an initial introduction of the day’s topic and student presentations on homework, students are provided guided activities to complete at their tables. Typically, the brief presentation (5 minutes) simply allows

the instructor to set the stage for the activities and the concepts for the day. The activities were those that accompanied the text, which was designed for future elementary teachers. Hands-on manipulatives are frequently used in the course, which is typically the first experience these students have had with using manipulatives in a math content course. While students are working together on activities, the instructor circulates around the room listening to students and their exchanges and gives gentle nudges when needed, rather than simply providing an answer. If a common question surfaces around the room, or there is a major point of confusion among several of the students, the groups may report out their work, and they may all work jointly to develop a solution. An active-learning classroom is one in which students feel comfortable failing, evaluating, and trying again (Miller-Reilly, 2007). An attitude of resilience and persistence in solving problems is the goal.

Students also would lead discussions of the topic at hand from the activities. During the activities, presenting a problem meant that a student would put his or her work up on the board after the class had sufficient time to work through the problem within their group. As time allowed the student who presented would explain the work.

Participation Grading

The active participation grade was a key element in what made the Number Sense course different from traditional mathematics classes. It was decided that students would be required to present several times before the end of the semester as part of their participation grade. The participation grade was more than just presentations. Students were expected to attend class regularly and attend at least two out-of-class mathematics activities. Activities have included helping grade problems in the High School Math contest, attending Cool Math Talks, and participating in UNO Math club events. These activities and events were selected to provide opportunities for future teachers to engage with others in rich mathematics and to take part in outreach activities within the mathematics education community at UNO.

Student Reactions to the Number Sense Course/Active Learning structure

The following are actual student comments after completing the Number Sense course, which helps to illustrate the impact on students. The results demonstrate how student beliefs about teaching and learning mathematics can change by taking even one course that employs the active learning method of teaching and focuses on the deep understanding of K–8 level mathematics.

This course was one of the hardest but most rewarding math classes I've ever taken. I know for me, having graduated in the late '90s, math was 80% procedure and 10% concepts. Needless to say, I always hated the two story problems at the end of the homework. Going into the semester, I had wanted to develop my understanding of math concepts. Math was something I always did well in when it came to procedures, but I needed to develop my math sense - to understand the why behind the what. I feel this class did that. I can see math in different ways that I never could before. Even my simple math sense such as adding quickly has improved. It's not to say this class wasn't at times frustrating. It was. I had to go back to the foundations of what I had learned decades ago to fill in the missing pieces. The hardest thing for me outside of the content was having to write specifically without seeing an example of it. Most everything was well explained verbally and in multiple ways, but I could have benefited greatly from those explanations being written down.

All this to say, I think this class is necessary for teachers. Knowing how to do math and teaching math are two very different things. This class helps teachers deepen their understanding of conceptual mathematics. I think I will be a better math teacher because I took this class.

Recruiting Future Teachers into the Noyce Math Scholarship Program

Before concluding, a look at the third point is given, which is perhaps of great interest with the expansion of Noyce scholarship programs nationally. Like many Track 1 Noyce programs, recruiting future teachers to get degrees in mathematics and to commit to teaching in high-need schools are two of the biggest issues. As students have been engaged in active learning classes, such as the Number Sense course, they seem to be more likely to apply for the Track 1 Noyce Scholarship program than those in traditional classes. This mimics what has happened in Calculus and Precalculus classes at UNO (Rech, Hodge, & Matthews, 2017). Every day, instructors get to hear students think and watch them work collaboratively with other students. Students are interacting and helping others in the classroom.

Faculty frequently encourage promising students to take part in a Noyce scholarship program as scholars and interns. Students who would significantly contribute to the Noyce program and potentially make future teacher leaders are identified. They are invited to: (a) apply to be a Noyce summer intern (an internship in teaching), (b) visit with our advisor to take more mathematics courses, (c) work with faculty on mathematics education research and other projects, and/or (d) work as learning assistants in mathematics courses.

The recruitment is a result of the efforts tied to the Actively Learning Mathematics RAC within the MTE-Partnership. A goal was to institutionalize active learning into the calculus sequence and use these teaching techniques as recruiting tools for future mathematics teachers. Although the institutional level of implementation has not been reached, the successes of using active learning in calculus with students and future teachers led to the expansion of using active learning to the Number Sense and History of Math classes. Promising teachers and future teacher leaders have become accustomed to and thrived while learning in an IBL format. They may be heavily influenced toward engaged and active teaching in future classes because of multiple IBL and active-learning experiences.

Future Plans and Conclusions

The Number Sense course appears to provide an opportunity for students to see many topics in mathematics from a new perspective. The students often reflect that it was “the most useful” math course they took as an undergraduate student and truly opened their eyes to what “deep understandings” in mathematics means. Future efforts could focus on how learning in this class affects their ability to understand mathematical concepts that are only taught in high school. Many of the students in active-learning classrooms have grown to think that active learning, which aligns with the mathematical practices of teaching, is an effective way to learn mathematics. Since we are asking our teachers to instruct this way in the classroom, we value this outcome. If Cox (2014) is correct, this leads us to believe that the future teachers in these active learning classrooms will also teach mathematics in an active manner. As a result, they will be better prepared to teach their students by directly incorporating the mathematical practices of teaching. Future efforts could study to what degree teachers who have significant active learning strategies modeled in their undergraduate curriculum use similar strategies in their own teaching.

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