Mathematical Modeling Modules: Curriculum Material for Secondary Teacher Education

Ricardo Cortez, Tulane University, rcortez@tulane.edu
Cynthia Anhalt, University of Arizona, canhalt@math.arizona.edu
Brynja Kohler, Utah State University, Brynja.Kohler@usu.edu
Watch the presentation: https://youtu.be/A9qWKokR340

Abstract

This NSF-funded project, the Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S^2)), supports collaborative development of mathematical modeling modules that were first piloted around the country in 2019–2020 and will be eventually made available widely. This report provides an overview of the structure of the material, highlights several modeling tasks, describes the use of simulations of practice in the curriculum materials, and summarizes activities conducted with piloting faculty.

Introduction

Mathematical modeling addresses complex social, economic, and scientific issues. Since the *Common Core State Standards of Mathematics* were issued about 10 years ago, modeling has gained prominence in the K–12 curriculum, even in states that did not adopt the Common Core. At the university level, mathematical modeling has been a course available at both the undergraduate and graduate levels. In fact, modeling is an area of research in the mathematical sciences in which one can earn a Ph.D. (e.g., Rochester Institute of Technology program). Yet, most teacher preparation programs do not require a mathematical modeling course, and in many cases, mathematics faculty are not familiar with approaches to teach modeling in K–12.

Mathematicians, engineers, social scientists, and many other people participate in mathematical modeling at the professional level in order to make predictions about the stock market, the weather, the efficacy of drugs under development, people's behavior, and many other situations. Modeling may be done to address a local problem in a community, to understand natural phenomena, or in response to a crisis like the coronavirus pandemic. In the last few months, we have been bombarded with mathematical models related to coronavirus showing that different models predict different outcomes depending on the assumptions made (NY Times, 2020). When more information about the spread of the disease became available, the model predictions showed increased agreement. The weekly updates of the models provided a real-time example of the modeling process at work.

The modeling process is essentially the same at all levels. The approaches and the models might be more complex or more sophisticated, but the process is essentially the same. Additionally, many communities of engineers, students, scientists, and others hold viewpoints on mathematical modeling dependent on how they experience and work with it. We propose that the development of mathematical modeling teaching practices in K–12 can benefit from acknowledging and including the different communities. Such collaboration can provide purpose and context for the tasks and give insights into the competencies that need to be developed.

This process has implications in teacher education. Mathematical modeling is not a prescriptive process; it is creative and personal. We believe that teacher educators can benefit from doing mathematical modeling to experience it the way students do, and from becoming familiar to some degree with the research on teaching and learning mathematical modeling. Both of these points provide appreciation for modeling and can inform ways in which teacher educators can guide pre-service teachers in the construction of their models and the modeling process itself.

Overview of Mathematical Modeling Course Materials

Our curricular materials include three modules in mathematical modeling for pre-service teachers (PSTs). Module 1, The Process and Purpose of Mathematical Modeling, incorporates lessons that focus on understanding the elements of the mathematical modeling process or cycle. Module 2, Incorporating Real Data in Mathematical Modeling, includes lessons in which prospective teachers consider data for creating mathematical models. Module 3, Diverse Perspectives in Mathematical Modeling, focuses on topics in which issues of equity and social justice arise, with mathematical modeling serving to provide insight within the topics. At the end of each module, the curricular materials require PSTs to write a reflection on the mathematical modeling process to capture growth in their learning across the three modules. At the end of the third module, a culminating final project provides the opportunity for PSTs to create their own modeling problem appropriate for the secondary curriculum.

The common components of each lesson include an overview with specific goals, relevant content standards, concepts that are beyond the standards, and materials such as handouts, slides, and pertinent readings for students. In addition, the lessons provide instructors with mathematical, historical, and pedagogical notes, as well as technological tool options. The lessons are structured with an introduction, including background information on a specific topic, guided exploration of a task, and opportunities for students to present their solutions. Each lesson incorporates sample approaches and possible models to aid instructor preparation.

Below we present a sample modeling task from Module 1 on water conservation. Included in the lessons are guiding questions for instructors to pose as PSTs work on the problem to serve as a catalyst to stimulate thinking.

Some say that showering uses less water than bathing. Others say that this is not true! Provide a method to determine if a shower or a bath uses more water and explain your approach. Keep in mind that older showerheads have a flow rate of up to 3.4 gallons/minute whereas energy-saving showerheads have a flow rate as low as 1.9 gallons/minute. Bathtubs also vary in size.	What information do we need? Where do we find this information? What do we do with information we need but we can't find? How do we develop a procedure to determine which is better? How do we present this procedure? What mathematics might be relevant to use? How do we know how accurate our results are?

Figure 1. Water conservation task and guiding questions.

In a Module 2 lesson, PSTs are prompted to come up with a model for the memorization process. Following a broader discussion of learning theory, they collect their own data to create, parameterize, and validate models that reflect their own experience and provide insight into a topic related to learning and teaching. A lesson in Module 3 addresses the reduction in Native American reservation lands. Examining several resources and historical documents, PSTs create models to determine areas of the Great Sioux Reservation in 1851 and 1876, as well as engage in discussions about the treatment of Native Americans in U.S. history.

Selected lessons in each module include a *simulation of practice* (SoP) activity that places prospective teachers in a position to plan class discussions and react or respond to students' thinking, either through a written or video reflection. These SoP activities are intended to engage PSTs in practices of teaching (Grossman et al., 2009) and serve as tools within the modules for the instructor to gather information and provide feedback on the prospective teachers' developing knowledge for teaching mathematical modeling. In a SoP associated with the water conservation task in Figure 1, we ask prospective teachers to describe how they would plan to facilitate an interactive discussion around this problem. The sample written SoP assignment (see Figure 2) includes a diagram of the mathematical modeling process.

Assume that your students have worked on the water conservation modeling task. In 1-2 pages, describe how you would plan to facilitate a discussion with your students which will allow you to elicit student thinking about the modeling process while addressing the following questions:

- What elements of the modeling process are prominent in this lesson? Explain your answer and give examples to support your argument.
- How would you extend the task to promote further iterations of the modeling cycle?

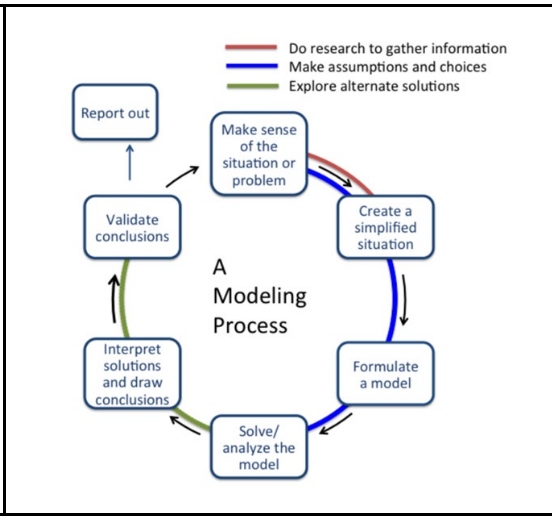

Figure 2. Example simulation of practice (SoP).

Our curricular materials include a rubric for instructors to provide feedback to the prospective teachers on their work in the SoP. Note that the descriptors in the rubric serve to help the prospective teachers and instructors focus on specific areas in the work of teaching mathematical modeling.

Table 1.
Rubric for Feedback to Prospective Teachers on the Simulations of Practice (SoP)

Descriptor	Meets Expectations	Does Not Meet Expectations
Central mathematical points	Central mathematical points are clearly defined.	Central mathematical points are missed or not clearly defined.
Appropriateness of questions	Questions posed help students advance their thinking.	No questions are included. Questions posed do not help students advance their thinking.
Relationship to modeling process	Discussion prompts address relationship to mathematical modeling process appropriately.	Discussion prompts do not address relationship to mathematical modeling process appropriately.
Advancing the understanding of the modeling process	The planned discussion is likely to advance student understanding of the modeling process.	The planned discussion is not focused and not likely to advance the modeling process.
Appropriateness of anticipated student responses	Anticipated student responses are appropriate.	Anticipated student responses are unreasonable, inappropriate, or not included.

Teaching with the Mathematical Modeling Materials

The first pilots of the Mathematical Modeling modules were conducted in the 2019–2020 academic year. The five instructors recruited to participate came from various institutions in the Midwest and South and planned to implement the materials in a mix of different courses: mathematical modeling for future high school teachers, methods of mathematics teaching, and mathematical content courses for future middle school teachers. To prepare instructors to teach with the materials, we conducted a summer session with a focus on developing the key instructional practices for mathematics teacher educators that the materials aim to promote for PSTs, namely: (1) generating questions and discussion that promote students' mathematical explorations; and (2) learning about student understanding using their explanations, justifications, and representations. To share the vision of these practices in undergraduate teaching we used readings from the MAA Instructional Practice Guide, and created videos of members of the MODULE(S^2) team teaching mathematical modeling lessons.

The structure of professional development activities during the summer provided instructors with opportunities to analyze and prepare several specific lessons. Research indicates that reflection on instruction and on one's students' performance is likely to result in change when observations are strongly linked to specific future instructional actions (e.g., Horn et al., 2015). We began by engaging in a mathematical modeling task from a particular lesson to delve into the mathematics. Then, we observed and collectively analyzed a video of a classroom session that highlighted a key instructional practice. Next, piloters would read the instructor guide, create their own plan, and conduct a lesson rehearsal with other participants acting as PSTs in a simulation. We video recorded and collectively analyzed these lesson rehearsals.

During the academic year, instructors taught their courses and collected data for the project. They received support and gave feedback through discussion boards on Canvas, and enactments of key lessons in each module were videotaped. We held monthly online professional learning community meetings over Zoom, to collectively analyze short clips from their lessons and discuss instruction and PSTs' performances on modeling tasks and simulations of practice.

Summary and Future Work

Mathematical modeling is in the unique position of being part of the K–12 curriculum, a topic in undergraduate mathematics, a technique used by professionals, and an active area of research. MODULE(S^2) has sought to develop materials that target future teachers' experience *doing* modeling along with mathematical knowledge for *teaching* modeling. In our research, we plan to analyze the SoP responses to gain insight into the PSTs' development of mathematical knowledge for teaching specific to modeling. Data from the pilots will inform our revision of student and instructor materials in preparation for the next round, planned for 2021–2022.

References

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: Theory and Practice, 15*(2), 273–289.

Horn, I. S., Kane, B. D., & Wilson, J. (2015). Making sense of student performance data: Data use logics and mathematics teachers' learning opportunities. *American Educational Research Journal, 52*(2), 208–242.