Mathematical Modeling Lesson Adaptation for Increasing Local Relevance

Will Tidwell, Utah State University, tidwell.ws@aggiemail.usu.edu
Brynja R. Kohler, Utah State University, brynja.kohler@usu.edu

Abstract

How can local issues be effectively integrated into a mathematical modeling situation, particularly in the development of curriculum intended for widespread adoption? This report summarizes qualitative improvements made to student and instructor materials of the MODULE(S^2)[1]: Mathematical Modeling for Secondary Teaching curriculum after implementation at the Utah State University Blanding Campus STEAM Expo—an event that provides enriching experiences in science, technology, engineering, arts, and mathematics every spring for the community in rural Southeastern Utah. At this event, we engaged in-service mathematics teachers in two lessons pertaining to water consumption and conservation from the mathematical modeling course. We gathered resources to take a contextualized approach to the water impoverished region of Southeastern Utah, and adapted lessons accordingly. To guide our work, we applied a Plan-Do-Study-Act Cycle and collected qualitative data from the instruction and interaction with participants to improve our materials.

Introduction

The *Common Core State Standards for Mathematics* (CCSS-M) defines mathematical modeling as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Mathematical modeling is thought to be a critical process in learning mathematics because it allows for real-world interaction with mathematics, making appropriate assumptions, and justifying and reporting results. Kartal, Dunya, Diefes-Dux, and Zawojewski (2016) assert mathematical modeling to be a necessary part of STEM professions and an influencer to show if students can think creatively about complex problems. With mathematical modeling shown as an important part of mathematics education today, teacher preparation programs should ensure proper instruction of these skills. Teachers' mathematical knowledge for teaching (MKT) predicts gains in student achievement, and general pedagogical knowledge is highly related to instructional quality (Ball, Hill, & Bass, 2005; König & Pflanzl, 2016). Well-designed and facilitated mathematical modeling activities have great potential for addressing MKT and general pedagogical knowledge.

Inside teacher education, research has shown that courses taught with mathematical modeling tasks lead to increases in mathematical modeling performance (Bal & Doğanay, 2014; Karaci Yasa & Karatas, 2018). Studies have shown that mathematical modeling performance has a positive relationship with mathematical modeling conceptions (Anhalt & Cortez, 2016; Son, Jung, & I, 2017). Mathematical modeling conceptions have been positively tied to effective mathematical modeling instruction (Son et al., 2017). Instructing teachers using mathematical modeling tasks can improve performance, conception of mathematical modeling, and effective mathematical modeling instruction.

[1] Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S^2)).

Smith, W. M., Callahan, K. M., Strayer, J. F., Jones, R. S., & Augustyn, L. C. (Eds.). (2019). *Proceedings of the eighth annual Mathematics Teacher Education Partnership conference*. Washington, DC: Association of Public and Land-grant Universities.

While mathematical modeling tasks are needed in teacher education, successful engagement in mathematical modeling requires that the tasks and their cultural contexts must have relevance. For this reason, we consider how mathematics and culture are linked, and aim to incorporate sociocultural modeling perspectives in this work. Sociocultural modeling is described as emphasizing “critical thinking about the role of mathematics in society, the role and nature of mathematical models, and the function of mathematical modeling in society” (Kaiser, 2017, p. 274). This idea of sociocultural modeling has close ties to D’Ambrosio’s (1985) *ethnomathematics*. Ethnomathematics focuses learning on the cultural experiences students have with mathematics to enhance understanding. Ethnomathematics utilizes students’ vast cultural background and experience in order to guide them through the modeling process. Sociocultural modeling emphasizes the idea of enculturation (Bishop, 1988) and students utilizing this knowledge of their culture to understand a problem with mathematics.

Problem and Purpose

Across the world, there are mathematical modeling prompts that are created for important local issues. Unfortunately, prompts may promote acculturation (Bishop, 1988) if not relevant to the culture in the area where these prompts are being used. Our goal is to improve the relevance of lessons for two water-related tasks by including local context and leading teacher participants to develop novel modeling tasks. Our specific research questions were:

1. What evidence of MKT development emerges during the implemented lessons from teachers in Southeastern Utah?
2. Do local teachers take away an appreciation for mathematical modeling, and see mathematical modeling as a productive approach for addressing water quality and water conservation issues?
3. What does this localization of a modeling task involve?

The implementation of the lessons involved in-service teachers, the participants. We, the researchers, believe that study results will address and inform the goals of preparing pre-service mathematics teachers. The in-service teachers’ knowledge of their students and their students’ cultures, interests, and motivations only enhance the study’s results for improving teacher preparation.

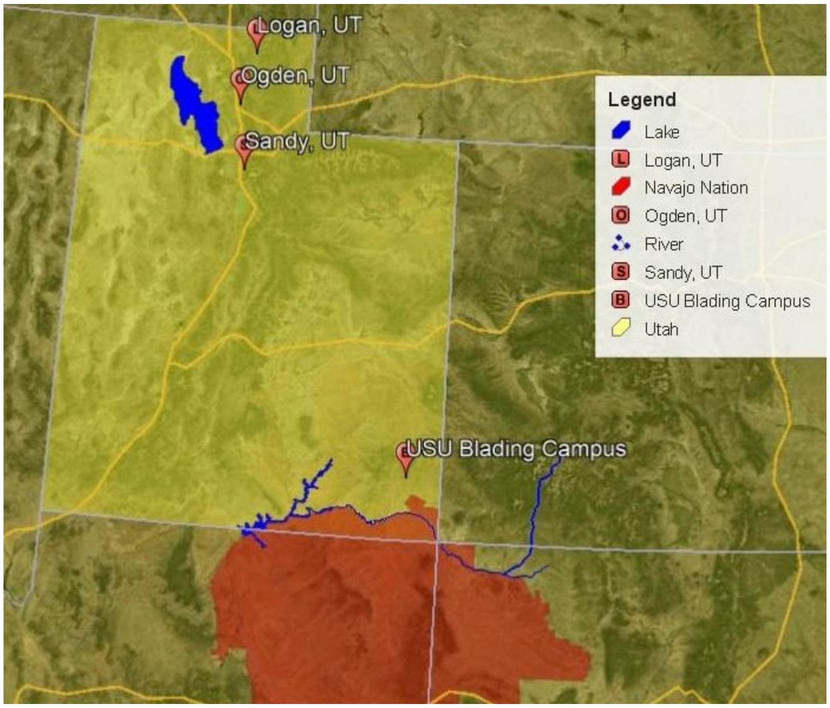


Figure 1. Map of Utah detailing the major locations pertaining to the study.

Conceptual Framework

A couple of viewpoints shaped the investigation of this study. First, the researchers used a Plan-Do-Study-Act (PDSA) Cycle developed by Deming (1993) to provide guided feedback for improvements throughout our investigation. The Mathematics Teacher Education Partnership's (MTE-Partnership) Research Action Clusters (RACs) frequently use this framework in curricular development studies.

Second, the researchers wanted to ensure local information about headlining issues were integrated into the workshop. Since the researchers were from an area in Northern Utah and the workshop was being held in Southern Utah (see Figure 1), we asked local collaborators for help gathering information and identifying speakers. A professional from USU Blanding pointed us toward numerous news articles that described various water-related issues in the area. We also connected with a local pre-service teacher from the Navajo Nation who shared some of her personal experiences regarding the effects of droughts, along with cultural and spiritual perspectives about water.

Third, the researchers applied the framework discussed in Aguirre, Anhalt, Cortez, Turner, and Simic-Muller (2019) that combines the process of mathematical modeling and discussions of social justice issues. After a broad social issue is addressed, a more specific situation can lend itself to mathematical modeling. The traditional cyclic mathematical modeling process is then engaged by making sense of the situation, researching information needed, making assumptions and choices, formulating and then solving a mathematical model, interpreting the solution, validating the outcome, and reporting results. After reporting the model's results re-contextualized in this specific social situation, participants build civic awareness and a sense of action. For example, our workshop used the Flint Water Task. The researchers introduced the social issue of water contamination and access to clean water in Flint, Michigan, and then raised the specific issue of water donations by several corporations. The participants then engaged in the modeling task, contextualized their solution to the social issue, and built their civic awareness and sense of action.

Additionally, our workshop was designed with equity-based teaching practices described by Aguirre, Mayfield-Ingram, and Martin (2013) to "strengthen mathematical learning and cultivate positive student mathematical identity" (p. 43). These equity-based teaching practices consist of going deep with mathematics, leveraging multiple mathematical competencies, affirming mathematics learners' identities, challenging spaces of marginality, and drawing on multiple resources of knowledge (Aguirre et al., 2013). The researchers framed this workshop as a collaboration. Our program included opportunities for participants to draw upon multiple resources, leverage multiple mathematical competencies, and go deep with mathematics. The workshop activities engaged participants in mathematical discourse and presented tasks with high cognitive demand with no set solutions and many solution strategies. The researchers recognized that a benefit of using mathematical modeling coupled with social issues is that multiple approaches emerge from the various backgrounds of participants.

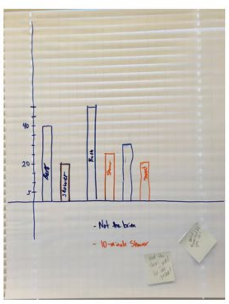
Methods

The framework for the methods is a PDSA Cycle for improvement. In this setup, the researchers extensively planned the professional development, constructed viable research questions, made predictions, and noted what data would need to be collected in order to answer questions (see Appendix A). The researchers looked at participant work samples, attended to discussions during the sessions, kept notes as presenters and observers, and collected information through pre- and post-surveys to answer the research questions highlighted in this study. In each of the sessions outlined in Figure 2, the researchers placed opportunities to look at participant work samples. In the first two sessions, the participant work consisted of posters made for gallery walks to report their results (see Figure 3). One person presented material while the other acted as an observer and recorded noteworthy observations. These notes collected were structured to look for mathematical and pedagogical connections, evidence of motivation and reflection, focus in the modeling process, and general observations on how the group work proceeded. The researchers constructed a pre- and post-survey to provide additional information to the discussions. They consisted of a combination of open-ended questions and questions with a Likert scale. The two surveys were administered at the beginning and end of the workshop (see Appendix B). The questions on both the pre- and post- survey were used to track potential evolution of conceptions of mathematical modeling, beliefs on modeling social issues, and comfortability of using mathematical modeling tasks in their classrooms. The pre-survey contains questions informed the researchers on the background of the participants. The post-survey contains questions helped the researchers plan for future workshops.

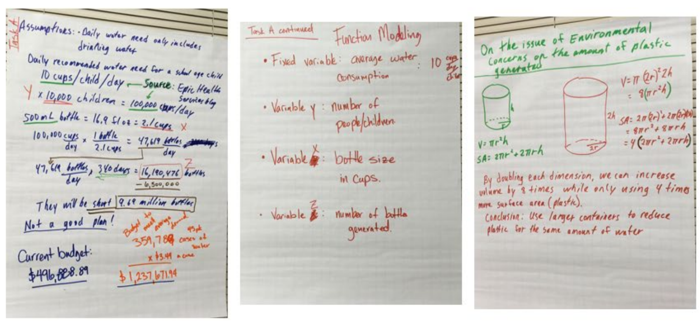
The Do phase of our PDSA Cycle included 17 workshop participants. In the Study phase, the researchers compiled and synthesized the data to answer research questions. Due to the qualitative nature of this study, researchers found relevant information in the observation notes to answer research questions. Surveys were interpreted in a similar manner for all open-ended responses. Participants' work was looked at for evolution of mathematical modeling practices and evidence of other areas. This data was used to answer our research questions and to direct future actions.

Workshop Plan
Water Conservation Session • Shower v. Bath Task involves analysis of whether it is more water efficient to take a shower or a bath. • Localizing Questions a. Where does your water come from? b. How much is water conservation an important concern locally?
Water Contamination Session • Flint Water Task Tasks involves assessing the amount of water bottles needed to support the youth of Flint for one year as well as eventually creating a strategy to lessen the environmental impacts by all of the plastic waste created. • Localizing Questions a. What are the clean water daily needs in your community? b. What other contamination issues have you heard of?
Local Water Issues Session • Introduce Cultural Perspective on Water in the Area • Present Summary of Headline Issues • Create Modeling tasks Participants create localize modeling tasks to fit the needs of their students.

Figure 2. Outline of workshop plan, showing the major tasks and activities.



Session 1



Session 2

Figure 3. The participants' work during the sessions consisted of posters made for gallery walks to report their results.

Findings and Discussion

Figure 4 shows a shift in comfort level from the pre- and post-surveys. More than three-fourths of the participants stated that they were either certain or almost certain they would use these materials in their own classrooms. Almost three-fourths of the participants self-reported only occasionally or rarely using culturally relevant word problems.

Three excerpts from the surveys showed appreciation for mathematical modeling, and its help in understanding social issues:

- "Mathematical modeling is difficult and time consuming to plan, but the outcome and affect on students seems to greatly justify the effort."
- "… mathematical modeling can begin to normalize and focus the conversation [about community issues]."
- "Models help to judge the validity of proposed solutions and their potential effectiveness."

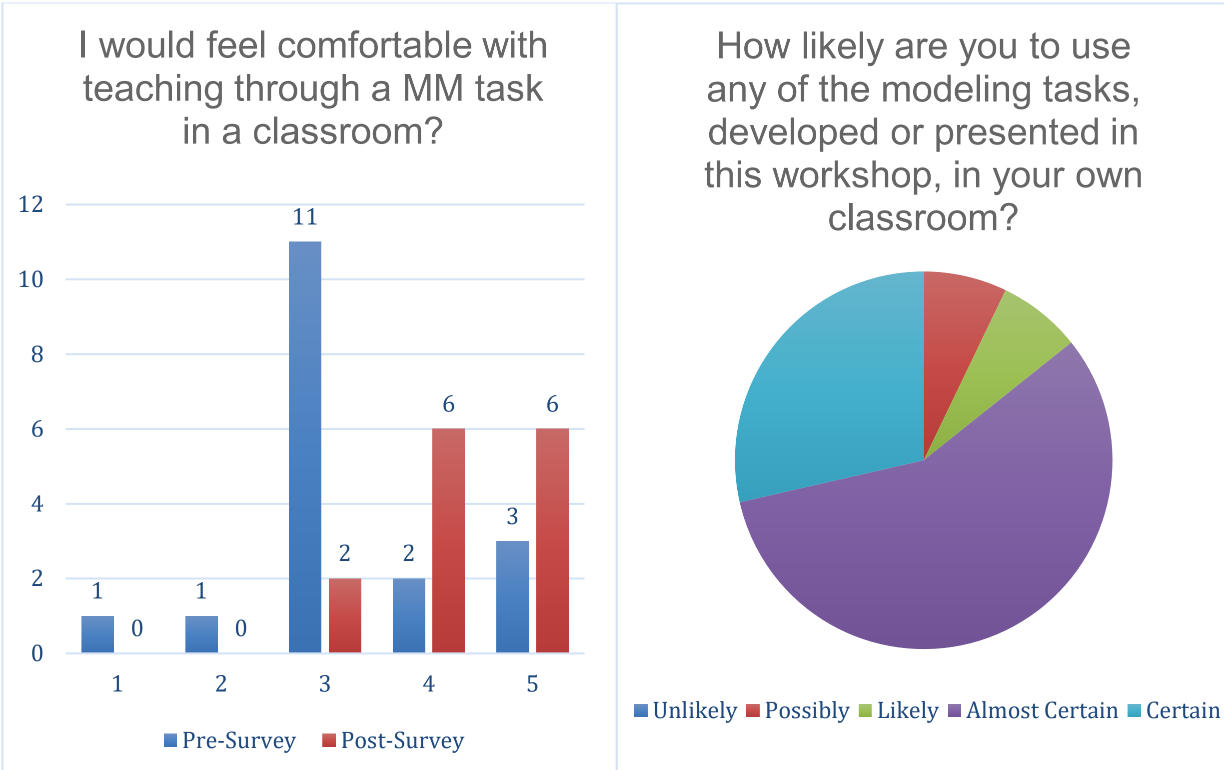


Figure 4. Survey questions regarding comfort level with modeling tasks.

The discussion of our findings also can be summarized by revisiting the three research questions:

1. *What evidence of MKT development emerges during the implemented lessons from teachers in Southeastern Utah?*

The evidence of MKT development during the implemented lessons included using mathematics in new ways. Participants used unit analysis, functions of two variables, scientific notation, surface area/volume, estimation, and error. For example, in order to discuss the surface area of a plastic bottle, the participants had to assume certain constraints over the bottle making it look like a can more than a bottle. Over the progression of the workshop, the evidence of tracking their work with gallery walks showed that the participants improved greatly in reporting their results to the group. They evolved from presenting simple calculations, tables, and charts to including assumptions, methods, sources, and conclusions. Participants showed increased emphasis on how choices were made for modeling with clearer articulation of assumptions.

2. *Do local teachers take away an appreciation for mathematical modeling, and see mathematical modeling as a productive approach for addressing water quality and water conservation issues?*

Evidence from the pre- and post-surveys indicated an increased awareness for water conservation and quality issues. In the pre-survey, one-third of the participants identified mathematical modeling to be useful in addressing social justice issues. Almost the entire group of participants did so in the post-survey. Additionally, the survey responses jumped from five responses dealing with water-related issues to 12 responses—showing that water conservation and quality were important social issues in their community. Lastly, participants showed growth in their appreciation and understanding of mathematical modeling. This evidence supports the research that performing mathematical modeling prompts will improve the participant appreciation of mathematical modeling (Anhalt & Cortez, 2016; Bal & Doğanay, 2014; Karaci Yasa & Karatas, 2018; Son, Jung, & I, 2017; Stohlmann, Maiorca, & Olson, 2015).

3. *What does this localization of a modeling task involve?*

Participants worked in groups to formulate mathematical modeling tasks of their own. They followed a four-step process when designing mathematical modeling tasks with local relevance. First, the participants made connections to people in the community with access to information and resources, which parallels the researchers' approach of contacting local experts in planning the workshop. Second, participants decided which information to use, choosing what they perceived as meaningful and engaging. Participants came up with tasks related to the following issues: mercury levels in fish, the geometry of recreational spaces, access of culinary water on the Navajo Nation, high levels of arsenic in well water, and the relationship between air pollution and automobile idle time—reflecting diversity in environmental and social issues beyond water-related topics. Deciding how to present the prompt was perhaps the most difficult aspect. Participants had to develop a question that would lead to multiple mathematical approaches. Third, participants sought more resources. They were already familiar with various sources of information, and searched the internet for reliable sources of additional facts and figures pertinent to their themes. Fourth, participants formalized and shared the modeling task and models they developed. They used a template document for mathematical modeling lessons provided by the researchers reflecting the general structure of lessons in the MODULE(S^2) Mathematical Modeling for Secondary Teaching course. Groups worked collaboratively to understand some different routes students might take when solving these problems. This fourth step helped in finding appropriate standards to align the modeling task to placement in curricula and appropriate implementation into the classroom.

Conclusion

We found that certain pedagogical techniques in the workshop helped develop mathematical modeling proficiency. When participants were able to see others' work and critique the work, this process helped them to structure reports of their own mathematical modeling work, and, ultimately, create novel mathematical modeling tasks. The researchers will add references to some specific water issues addressed in this workshop to increase awareness about native lands in Southeastern Utah. The materials also will be revised to include tools and suggestions in the lessons for how to increase the relevance for the community in which the lessons are implemented. Since multiple participants noted that the whole-group discussions about the community issues were just as important as doing the mathematical modeling, lesson plans within materials for pre-service mathematics teachers will include more time for discussing the social issues.

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Appendix A: The PDSA Cycle Overview

1) PLAN		
Questions:		
What evidence of MKT development emerges during the implemented lessons from teachers in Southeastern Utah?	Do local teachers take away an appreciation for mathematical modeling, and see mathematical modeling as a resource for addressing water quality and water conservation issues?	What does this localization of a modeling task actually involve? How much work is it to adopt materials to make them relevant for a specific community?
Predictions:		
We expect to see application of proportional reasoning, algebraic representations of functions, attention to units of measurement and conversions. We also expect to see growing sophistication in participant understanding of math modeling over the course of the workshop.	We expect enthusiasm for mathematical modeling as a means for addressing community issues to increase. We do not know whether participant willingness to teach through mathematical modeling activities will change.	Our planning for the workshop included the following stages: -Making connections to people in the community with access to information and resources. -Deciding how and what to present. (Our choices were based on anticipation of what may be impactful to the audience. We need to assess how effective these choices were.) -Creating ways for participants to seek and investigate resources on their own and make use of the resources we provided.
Data:		
Posters for mathematical modeling work and observing participant discussions. Presenter notes.	Open-ended survey with questions pertaining to this. Discussions and presenter reflections. Possibly follow up after the workshop.	Materials from sessions. Tasks and lesson plans generated by participants. Questions/discussion from after each session. Presenter notes and reflections.

2) DO
Recording data during the sessions was a challenge, since so much was going on, it was hard to parse the essential information. On the other hand, that richness of the data added to the depth of the results and the experience for the people involved. The audience was very enthused about the conversations that occurred regarding social and political issues that were raised through the modeling task. The questions raised and the discussions that arose motivated the participants who felt these discussions were just as important as doing the mathematical work in the task. The only obstacle that we dealt with was time—however, we did have sufficient time to get the plans started. Preparing a complete set of mathematical tasks would take a longer amount of time.

3) STUDY		
What were the results?		
Mathematics used by participants: Unit analysis, functions of two variables, scientific notation, surface area/volume, estimation and error. Reporting on modeling changed from showing simple calculations/tables/charts toward showing assumptions, calculations/tables/charts, methods, and conclusions. Discussions increasingly emphasized how choices were made for modeling with clearer articulation of assumptions.	The survey showed great evidence, as did discussions with participants. Participants were very productive in finding and sharing resources, but needed more time to develop the modeling tasks in detail. They liked having a short template to follow. Participants gained appreciation for social-cultural perspective of mathematical modeling.	They did make connections to their local water resources in their communities. On the question, "Where does your water come from?" we got everything from "I have no idea," to various sources including wells, reservoirs, etc. After participants made connections, they grappled with deciding on what issues would be personally meaningful to their students. They created lessons on the following topics: Mercury levels in fish, Geometry of recreational space, Culinary water access in Montezuma Creek, Arsenic in well water, Air pollution and automobile idle time. This variety surprised us; clearly it wasn't just limited to water issues.

4) ACT
What changes will we make to the MODULE(S^2) lessons? Include tools and suggestions for how to increase the relevance for the community near you. Raise awareness about water issues on native lands in Southeastern Utah. Include descriptions of how the gallery walk procedure will benefit the discussion and growth of reporting mathematical modeling results as the MODULE(S^2) lessons are implemented. What is our summary of steps involved in creating meaningful tasks for addressing community issues locally? 1. Making connections to people in the community with access to information and resources. 2. Deciding how and what to present. (Our choices were based on anticipation of what may be impactful to the audience.) 3. Creating ways for participants to seek and investigate resources on their own and make use of the resources we provided. 4. Formalizing and further sharing the participant-generated tasks.

Appendix B: Survey Instrument

Survey Questions

Both Pre- and Post-Survey	Pre-Survey Only	Post-Survey Only
What does mathematical modeling mean to you? Think about issues that are going on in your community, what are some community or social issues that are important to you? What is the value of mathematical modeling when addressing community issues? I would feel comfortable with teaching through a mathematical modeling task in a classroom? 1-5	How frequently do you use mathematical modeling tasks in your teaching? How frequently do you use culturally relevant word problems in class? Both with options of Never, Rarely, Occasionally, Frequently, and All of the Time	What are your takeaways from this workshop? What will you remember and use after this workshop? How likely are you to use any of the modeling tasks, developed or presented in this workshop, in your own classroom? With options given as Unlikely, Possibly, Likely, Almost Certain, and Certain.