Developing a Framework for Equitable Mathematics Instruction

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Active learning and inquiry based instructional practices, as enacted in university Precalculus through Calculus 2 courses (referred to as the P2C2 sequence), have been shown to be beneficial in numerous ways. Freeman et al. (2014) found that active learning was associated with increased exam scores and concept inventory results, as well as significantly decreased withdraw and failure rates to a degree that calls into question whether traditional lecturing should continue to be used as a control in research studies at all and that supports "active learning as the preferred, empirically validated teaching practice" (p. 8410). Inquiry-based learning (IBL) in college mathematics has been shown to result in benefits described as learning gains in cognitive, affective and collaborative areas, for both women and men, while also decreasing the gap between the genders (Laursen, Hassi, Kogan, & Weston, 2014).

However, students who are members of underrepresented groups in mathematics, including but not limited to women and students of color, describe their experiences in active learning P2C2 courses as less inclusive, less positive and more marginalizing as compared to their white male peers (Voigt, 2017). Rasmussen and Ellis (2013) found that, within institutions that were part of the *Characteristics of Successful Calculus Programs* study, students who planned to continue on to a second semester of calculus, but changed their minds and stopped taking calculus after one semester, included significantly more women than men. While women make up approximately half the students in college Calculus 1 courses, their numbers drop off precipitously after the first semester of calculus (Rasmussen & Ellis, 2013). Students of color and students who are poor often face a different set of challenges in that they are more likely to experience high-school learning environments that are under resourced and that may not provide access to high-level mathematics courses (Berry, 2015). These learning experiences can lead to placement into precalculus rather than calculus in their first semester in college, and this scenario places an additional barrier to continuing in mathematics or mathematical fields of study. Responding to experiences of students of color in undergraduate mathematics courses Jett (2013) cites Leonard & Martin (2013) in his call for college mathematics instructors to take up culturally responsive pedagogy to "enact the brilliance that African American students bring to the mathematics space" (p. 102). Taken together, the research and evidence lay out a clear call for further investigation into the nature and characteristics of mathematics instruction that supports positive and inclusive learning experiences in undergraduate P2C2 courses, particularly those students who are members of underrepresented groups in mathematics.

This paper presents a theoretical model for equitable mathematics instruction, which I consider to be instruction that supports more positive and inclusive learning experiences for all students, especially for students who are members of underrepresented groups in mathematics. This framework for equitable mathematics instruction is based on a review of research in mathematics education that demonstrates the critical role of identity as a doer of mathematics in contributing to students' decisions to participate in mathematics (e.g., Boaler & Greeno, 2000), while also showing the degree to which students who are members of underrepresented groups in mathematics frequently persist in spite of marginalizing racialized and/or gendered experiences in mathematics communities (e.g., Joseph et al., 2017; McGee, 2015; e.g., Solomon, Radovic, & Black, 2016). Research on Black women and girls' experiences in mathematics has described robust mathematics identity as "an aspect of self-

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actualization that is needed for persistence, engagement and sustained success" in mathematics (Joseph, Hailu, & Boston, 2017, p. 203). McGee's (2015) research on the experiences of high-achieving Black college students defines robust in the context of mathematics identity as "the strength and agency that students develop in spite of their racialization to maintain self-motivated mathematics success" (p. 604).

The framework put forth in this paper suggests that mathematics instruction that values students' personal identities, enabling them to enact their fully intact cultural, racial and gender identities while doing mathematics, may support sense of belonging in mathematics. The model is built on the premise, which is in alignment with mathematics identity research, that a student who develops an identity as a doer of mathematics and a sense of belonging in mathematics in concert with each other – i.e., what I refer to in this paper as a robust mathematics identity – may be more likely to persist in mathematics. Equitable mathematics instruction is thus conceptualized as instructional practices and instructor beliefs that cultivate positive and inclusive mathematics learning environments that support students, particularly those who are members of underrepresented groups in mathematics, to develop robust mathematics identities and to persist in mathematics because of, rather than in spite of, their experiences learning and doing mathematics.

Theoretical Perspective

Learning is a social process (Wenger, 1998) and students' learning in mathematics classrooms is highly influenced by their ways of "engaging in and contributing to the practices of their communities" (Wenger, 1998, p. 7). Students who become participants (Wenger, 1998, pp. 55-56) within their mathematics learning communities take up a form of learning that has been shown to support their ability to envision themselves pursuing further study in mathematics or other STEM fields (Boaler & Greeno, 2000; Cobb & Hodge, 2002). Students' development of an identity as a doer of mathematics depends on the development of a vision of themselves doing mathematics that aligns with their perception of what it means or involves to do mathematics (Boaler & Greeno, 2000). In other words, students who experience roles as central participants in mathematics learning communities, and whose experiences of doing mathematics in those communities align with their visions of themselves as people, have been shown to be more likely to develop identities as doers of mathematics and to persist in mathematics or STEM fields of study.

Boaler & Greeno (2000), citing Holland, Lachicotte, Skinner, & Cain (1998) state that they are building on the theory that "identities develop in and through social practice" and use the term "'positional identity' to refer to the way in which people comprehend and enact their positions in the worlds in which they live" (p. 173). This attention to people's positions in the world points to the importance of drawing from research on the sociopolitical nature of mathematics and mathematics learning experiences (Gutiérrez, 2013; Nasir & McKinney de Royston, 2013; Valero, 2004). Aguirre et al. (2017) state that "a sociopolitical approach allows us to see the historical legacy of mathematics as a tool of oppression as well as a product of our humanity" (p. 125). Research demonstrates that members of certain populations (e.g., students of color and women) experience reduced opportunities to participate in the learning and doing of mathematics due to structures that are systematic and pervasive (e.g., Berry, Ellis, & Hughes, 2014; Langer-Osuna, 2011). Taking a sociopolitical perspective enables us to attend to the ways that "people's positions in the worlds in which they live" (Boaler & Greeno, 2000) are related to their positions in the mathematics worlds in which they take part.

What is Equitable Mathematics Instruction?

Equitable mathematics instruction refers to classroom practices—including instructional practices and instructor beliefs—that lead to development of students' robust mathematics identities through the cultivation of

learning environments that support: (a) identity as a doer of mathematics and (b) a sense of belonging in mathematics. Robust mathematics identities develop when students' identity as a doer of mathematics and sense of belonging in mathematics are both fully supported by their experiences in mathematics learning communities. This paper theorizes that active and inquiry-based instruction, enacted in ways that are student centered and genuinely focus on students' ideas, experiences and ways of understanding mathematics, are more likely to support students' positive identities as doers of mathematics. My research further suggests that instructors' focus on students' assets or strengths, and instructors' awareness of mathematics education as sociopolitical, are likely to support students' sense of belonging in mathematics. Taken individually each of these components may function as valuable improvements to instruction, but they may or may not be enacted in ways that are entirely equitable or that lead to improved outcomes for students who are members of underrepresented groups in mathematics. When these instructional practices and beliefs are enacted together in complementary ways, they may represent a powerful and reliable model of equitable mathematics instruction.

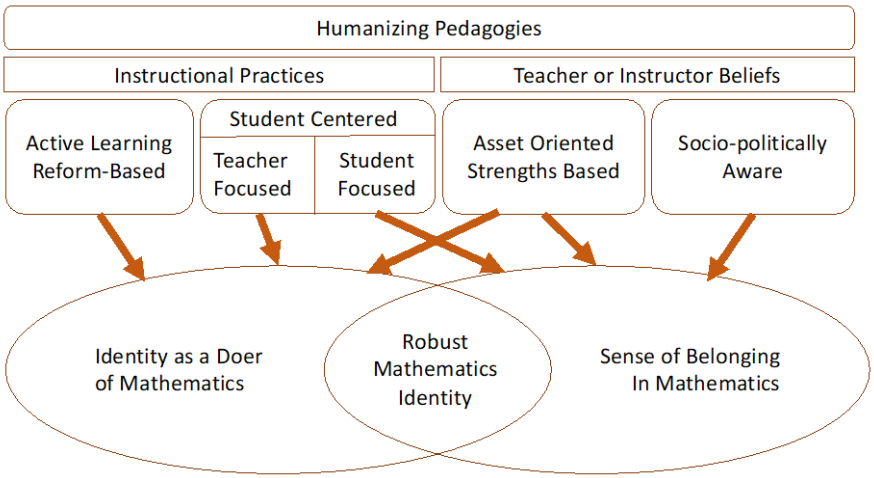

Figure 1. A conceptual model of equitable mathematics instruction.

Supporting Students' Identities as Doers of Mathematics

A student's identity as a doer of mathematics depends on their development of a vision of themselves doing mathematics that aligns with their perception of what doing mathematics means or requires (Boaler & Greeno, 2000). In other words, students who take on roles as central participants (Wenger, 1998) in mathematics learning communities, and whose experiences of doing mathematics in those communities align with their visions of themselves as people, are more likely to develop identities as doers of mathematics and to persist in mathematics or STEM fields of study. Instructional practices that support development of identities as doers of mathematics do so by supporting students' as sense-makers and problem-solvers in mathematics learning communities. Instructor beliefs that encompass an asset orientation, or that are strengths-based, may manifest as instructors' ability to take up instructional practices that anticipate student success with complex mathematical sense-making and problem solving.

Active and inquiry-based learning. Design principles for active learning were developed by the Active Learning Mathematics Research Action Cluster (ALM RAC) of the Mathematics Teacher Education Partnership (MTE-Partnership) and reflect a synthesis of research on active and inquiry-based learning. These principles encompass instructional practices that engage students in active meaning-making and sense-making and that support students to share "partially developed conjectures, explanations and representations of solution

strategies" (Webb, 2016, p. 2). Active and inquiry-based learning have been shown to support students' development of identities as doers of mathematics (e.g., Boaler & Greeno, 2000) and contribute to increased success and persistence in university mathematics (Freeman et al., 2014; Laursen et al., 2014).

Student-centered instruction. Introduction of the terms "teacher-focused" and "student-focused" allow us to draw a distinction and intentionally highlight differences between these two forms of student-centered instruction. The term "student-centered" refers to learning environments in which students' voices are heard more than the instructor's voice and/or students are actively participating in the doing of mathematics. However, student-centered instruction that is teacher-focused allows for and frequently refers to learning environments in which the students are at the center of activity, but the focus remains on the teachers' ideas, understandings, and methods. Questions, problems, and solution strategies are still conceived of and posed by teachers. This paper uses the term "student-focused" specifically to describe contexts in which students are participating in doing mathematics in ways that begin with focusing on their mathematical ideas and understandings and proceed by valuing their perspectives, experiences, and relevant issues in their lives. In student-focused instructional contexts, the students have a meaningful degree of agency over the questions, problems, and solution strategies.

Supporting Sense of Belonging in Mathematics

Gutiérrez (2018) states "beyond being seen as a legitimate participant (a "doer" of mathematics), a student should be able to feel whole as a person—to draw upon all of their cultural and linguistic resources—while participating in school mathematics" (p. 1). Bringing this feeling to fruition requires that mathematics learning environments support both students' development of identities as doers of mathematics and also their ability to "feel whole as a person" in mathematics—i.e., their sense of belonging in mathematics. Instructors whose belief systems are asset-oriented or strengths-based and reflect sociopolitical awareness may be well positioned to interact with students in ways that facilitate students' development of a sense of belonging in mathematics. Instructional practices that center and build off of students' ideas and experiences, i.e., those that are student-focused, may further contribute to students' sense of belonging in mathematics.

Asset or strengths-based orientations. In order to clarify what is meant by the terms "asset orientation" and "strengths-based," this paper first describes the nature of deficit thinking, which is more prevalent in society and education, and which is particularly strong in mathematics. The term "deficit orientation" refers to ways of viewing students' knowledge, learning abilities, accomplishments, communities and/or students themselves that are focused primarily on students' shortcomings,or are predominantly concerned with what students do not know, seem unable to learn, or are unable to do, as well as resources that are unavailable in communities or the ways in which the students themselves seem to fall short of some ideal. Jett (2013) states that "deficit-oriented ideological paradigms and treatises, such as achievement gap discourse, often frame students of color, particularly African Americans, as mathematically deficient" (p. 103).

In contrast, an asset orientation, or a strengths-based view of students, is one that attends closely to identifying what students *do* know, *can* learn, and *can* do, as well as what resources *are* available in their communities and more generally the strengths and assets that the students possess. An asset orientation is likely to inform interactions that support a greater student sense that their mathematical ideas and experiences are valued and that their personal identities are fully welcomed and appreciated in mathematics classrooms.

Sociopolitical awareness. "People's positions in the worlds in which they live" (Boaler & Greeno, 2000) are related to their positions in the mathematics worlds in which they take part, and social contexts have been shown to impact the ways that people learn and do mathematics (Nasir, Hand, & Taylor, 2008). This attention to people's positions in the world points to the importance of attending to research on the sociopolitical nature of mathematics and mathematics learning experiences (Gutiérrez, 2013; Nasir & McKinney de Royston, 2013; Valero,

2004). Aguirre et al. (2017) state that "a sociopolitical approach allows us to see the historical legacy of mathematics as a tool of oppression as well as a product of our humanity" (p. 125). Sociopolitical awareness is likely to enable instructors to attend to the realities of how students' experiences, ways of being, and interactions in society impact their experiences, ways of being, and interactions in mathematics classrooms. This awareness extends to instructors being aware of factors that originate outside the mathematics classroom that may affect the ways that students choose to participate in mathematics classrooms. This research suspects that sociopolitical awareness may serve to help instructors broaden their conceptions of what are accepted, desirable ways of participating in mathematics classrooms.

Furthermore, with regard to language in mathematics classrooms this paper claims that there is a need to broaden what are considered to be valid ways of communicating about mathematics—i.e., to expand what is valued as the "language of mathematics"—and that this represents a crucial component of validating students' own ideas, experiences, ways of understanding, and ways of communicating about mathematics. In other words, broadening the nature of accepted communication in mathematics classrooms may be one way of enacting instruction that is student-focused and asset-oriented. Furthermore, for bilingual students in particular, inviting, supporting and encouraging students to make sense of mathematics and share their personal experiences with mathematics using the full range of their linguistic resources (i.e., translanguaging) is necessary to students' development of a sense of belonging (i.e., their ability to be fully themselves) in mathematics communities.

Supporting Students' Development of Robust Mathematics Identities

As previously described, identity as a doer of mathematics is crucial to students' decisions to persist in mathematics. However, research describing the experiences of Black women and men who have persisted in mathematics (e.g., Joseph et al., 2017; McGee, 2015) makes clear that marginalized students are persisting in spite of racialized and gendered experiences in mathematics learning communities. It is possible that experiences that center students' personal identities, enabling them to participate in mathematics while also bringing their full cultural, racial, and gender identities into those mathematics spaces, may support sense of belonging in mathematics. A student who develops an identity as a doer of mathematics and a sense of belonging in mathematics in conjunction with each other, i.e., a robust mathematics identity, may be more likely to persist in mathematics because of, rather than in spite of, their experiences learning and doing mathematics.

Envisioning Equitable Mathematics Instruction in Action

It may be easier to imagine equitable mathematics instruction through consideration of three scenarios. In each scenario a potential instructor response is first described in a way that is quite common in mathematics classrooms, but which may fall short of supporting all students to engage positively with that particular learning opportunity. This paper then offers a contrasting alternative response that may be more likely to support the student's identity as a doer of mathematics and sense of belonging in mathematics, i.e., their robust mathematics identity, by reflecting an asset orientation or sociopolitical awareness or by enacting student-focused instruction.

<u>Scenario 1</u>: A student contributes an idea or a strategy for solving a mathematical problem in class. As described the strategy is not likely to produce a correct solution to the problem.

- The instructor might identify misconceptions or misunderstandings and correct them. This approach focuses on what is incorrect about the student's ideas, and then positions the instructor's way of thinking about the relevant concepts at the center of the conversation.
- Alternatively, the instructor could identify and acknowledge what aspects of the student's suggestion are likely to be useful or productive and facilitate learning that builds off of those ideas. This instructional approach positions the student's ideas as valuable, demonstrating an asset orientation toward the

students' mathematical ideas. It also keeps student thinking at the center of the classroom conversation reflecting a student focused approach.

Scenario 2: A student describes a mathematical idea using linguistic practices that differ from those of the instructor, perhaps using words and phrases from a language other than English or speaking in a dialect that is associated with a particular race, economic class, or geographic location.

- The instructor might respond less positively to this student's idea, or quickly move on to another student, because the structure or nature of the language practices are unfamiliar. This sends the message, intended or unintended, that this student's ideas aren't worthy of further consideration, or that it is not worth the time to come to clarity about the mathematical nature of these ideas. This approach may reflect a deficit orientation toward the mathematical ideas of students whose language practices differ from those of the instructor, or a lack of sociopolitical awareness about the significance of different language practices. Students whose ideas are responded to in this way lose the opportunity to access roles of central participation in the mathematics classroom.
- Alternatively, the instructor could consider this student's idea to be just as valid as others coming from students whose language practices are more similar to those of the instructor. The instructor may need to spend time and/or effort ensuring the student's idea is understood and acknowledged. That time reflects an asset orientation toward the student's mathematical thinking and a sociopolitical awareness about the use of language to express ideas.

Scenario 3: A student shares a problem-solving strategy that is unconventional or unfamiliar to the instructor.

- The instructor could dismiss the suggestion because they are unfamiliar with the student's approach. This instructional approach might be reflective of a deficit orientation based on an assumption that the idea has limited merit, or it could be related to lack of sociopolitical awareness of differing ways of communicating about and/or doing mathematics.
- Alternatively, the instructor might support the community to engage with and explore the idea to reveal its value and relevance. This strategy draws upon an asset orientation by positioning the idea as important and useful, and might also indicate sociopolitical awareness of how students' varied experiences in life can influence their ways of thinking about and doing mathematics.

Finally, a fourth example of how instructors can pose open-ended questions that open space in the classroom conversation for students to interact with mathematics in ways that make the most sense to them, can be achieved by simply changing the word "the" to "a":

Scenario 4: An instructor wishes to ask students to graph a relationship.

- This task could be presented as, "draw **the** graph," implying that there is one correct answer. Presumably the correct answer will reflect a teacher -centered approach since it will be the one the instructor has already thought of or the one the instructor deems to make the most sense.
- Alternatively, the instructor could ask students to "draw **a** graph" to invite a broader range of responses that reflect students' ideas. The answers may vary somewhat based on the ways that students set up their graphs, their choice of units, or their decisions about what dimensions to display on each axis. It's also likely that students who understand the scenario presented and the mechanics of graphing will provide graphs that are mathematically accurate and that reflect their own way of thinking about important characteristics of the scenario. This approach reflects a student-focused approach since students are not required to reproduce the instructor's way of thinking about the mathematics in order to earn credit for their work.

In each of these cases the alternative approach represents a way of enacting active learning that is not only student-centered but also student-focused, that reflects an asset orientation to students' ideas and contributions, and that demonstrates an awareness of the ways that sociopolitical factors can influence student participation in mathematics classrooms. Each of these examples reflects the instructor's genuine interest in understanding the student's contributions as well as use of instructional practices that model the importance of valuing and taking up students' ideas.

Conclusion

Successful implementation of equitable mathematics instruction depends partially on the affordances of mathematics department contexts that provide especially supportive structures and exhibit particular community characteristics, which include structural supports and community influences (e.g., tutoring centers, strong personal relationships, and curriculum adjustments that meet the needs of particular populations of students). Research has shown that these community structures play an important role in supporting Black women and girls, in particular, to develop robust mathematics identities and to persist in mathematics (Joseph et al., 2017).

The proposed model of equitable mathematics instruction contributes to the goal of increasing our understanding of how mathematics instruction might embrace and value students' full cultural, racial and gender identities in mathematics learning spaces. In so doing equitable mathematics instruction may play an important role in rehumanizing mathematics, mathematics instruction and mathematics communities. It is theorized that equitable mathematics instruction, through supporting development of robust mathematics identities that include both identities as doers of mathematics and sense of belonging in mathematics, may contribute to increased participation among students who are members of persistently underrepresented and historically marginalized groups in mathematics.

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