Opportunities Presented by Mathematics Textbooks for Prospective Teachers to Learn to Use Mathematics in Teaching

Yvonne Lai, University of Nebraska-Lincoln, yvonnexlai@unl.edu
Cody L. Patterson, University of Texas at San Antonio, cody.patterson@utsa.edu

Abstract

Secondary teachers have been documented to find content courses ineffective at developing instructional practices for high school teaching. We examined one potential contributor to this perception: tasks in textbooks for mathematics courses designed for prospective secondary teachers. We analyzed commonly used textbooks for whether and how their tasks situated mathematics in teaching scenarios. We found low percentages of such tasks in chapters addressing functions, expressions, and equations. For comparison, we analyzed a chapter on fractions in a textbook for prospective elementary mathematics teachers, finding that such tasks constituted almost half the available tasks. We find that one way to provide more opportunities for prospective secondary teachers to practice using knowledge in the context of teaching is to take existing tasks, which may be general in scope, and embed specific examples within them. We argue that doing so would increase both the quantity and the quality of opportunities for pre-service teachers to learn to use mathematics in teaching.

Keywords: mathematics content courses, secondary teacher education

Introduction

Teacher education – particularly mathematics coursework within a prospective teacher's preparation program – presents a special opportunity to learn to leverage mathematical knowledge for making teaching decisions. As scholars have argued, mathematical knowledge for teaching is a form of applied mathematics (e.g., Bass, 2005); and this form of knowledge has a greater impact on quality of teaching and opportunities for student learning than "purer" forms of subject matter knowledge (Rockoff, Jacob, Kane, & Straiger, 2011; Baumert et al., 2010).

However, pre-service and in-service secondary teachers have been documented to find content courses ineffective at developing instructional practices for high school teaching. Research suggests two main reasons: the content of undergraduate mathematics seems irrelevant to secondary teaching, and the norms of discourse used in academic mathematics seem inapplicable (e.g., Moreira & David, 2008; Ticknor, 2012; Wasserman et al., 2015). Even if content courses are in fact designed to address content, norms, and skills that are useful for teaching, teachers are unlikely to draw on resources they consider irrelevant. Thus, these findings suggest that improving the status quo involves understanding the opportunities that prospective secondary teachers do have to apply mathematics to teaching in their pre-service work and to what extent these opportunities are authentic to teaching practice.

In this paper, we analyzed opportunities for applying mathematics to teaching in a particular resource: tasks in textbooks for courses designed for prospective secondary teachers. We focused on a particular content cluster within these textbooks: functions, expressions, and equations. We asked: (1) To what extent do tasks apply

mathematics to teaching situations? and (2) What is the nature of how tasks situate mathematics in teaching? We anticipate that our findings may inform the work of the MODULE(S)^2 Research Action Cluster of the Mathematics Teacher Education Partnership (MTE-Partnership).

We found that tasks that apply mathematics to teaching situations are relatively rare – so much so that we conducted a comparison analysis of a popular textbook for prospective elementary teachers. We found that not only were tasks applying mathematics to teaching far more common in the elementary content textbook, but also that the tasks differed in nature, and were arguably closer to teaching practice. We report the results of the analysis for textbooks for secondary and elementary levels, and we argue that the contrast in these texts reveals features that are consequential to designing tasks that are authentic to teaching.

Conceptual Framework

We use *tasks* to refer to activities in textbooks designed by the authors for learners to do. Because tasks focus learners' attention on certain aspects of content and on certain ways of processing and reporting information (Doyle, 1983), tasks in textbooks for prospective mathematics teachers have the potential to direct attention to how mathematics can be used – and is useful – in teaching. We interpret tasks as potential *approximations of practice*, "opportunities for novices to engage in practices that are more or less proximal to the practices of a profession" (Grossman et al., 2009, p. 2058). Approximations of practice play the critical role in teacher education of providing opportunities for reflection on professional practices and judgments that may in actual teaching require extemporaneous thinking, as well as "deliberate practice" (Ericsson, 2002) of recurrent work of teaching.

Data

The data for this study are 715 tasks in four textbooks. Three textbooks were designed for use in mathematics courses for prospective secondary teachers and one textbook for prospective elementary teachers. Table 1 summarizes the data.

Table 1

Textbooks Analyzed

For prospective secondary teachers	Chapters analyzed	# Tasks
Usiskin et al. (2002)	Ch. 3 (Functions), Ch. 4 (Eqns.)	198
Bremigan, Bremigan, & Lorch (2011)	Ch. 1 (Functions), Ch. 2, 3, 12 (Eqn. Solving)	270
Sultan & Artzt (2010)	Ch. 3 (Eqns.), Ch. 9 (Functions & Modeling)	129
Conway (2010)	No chapters specifically focusing on functions, expressions, or equations	0
For prospective elementary teachers		
Beckmann (2011)	Ch. 2 (Fractions)	118
Total tasks		715

Selection of textbooks. In their review of capstone courses for prospective secondary mathematics teachers, Cox, Chesler, Beisiegel, Kenney, Newton, and Stone (2013) suggested that the most commonly used textbooks are the four listed in Table 1. Additionally, a search of textbooks published by the Mathematical Association of America, the professional society of mathematicians that focuses on mathematics accessible to undergraduates, found one textbook for secondary content courses, Bremigan, Bremigan, and Lorch (2011). Beckmann's (2011) book was chosen for its high score in number and operation (including treatment of fractions)

in the National Council of Teaching Quality's (2008) report *No Common Denominator.* There is no comparable survey of textbooks for prospective secondary mathematics teachers.

Selection of chapters*.* In our analysis of texts for prospective secondary teachers, we selected chapters on functions and equations, since the learning and teaching of these topics have been relatively well-researched (e.g., Knuth et al., 2006; Oehrtman, Carlson, & Thompson, 2008). We reasoned that such topics were more likely to afford tasks that applied mathematics to teaching, as more research has been conducted on these topics with regard to teaching practices, student conceptions, and teacher conceptions. For comparison, we selected the chapter on fractions in Beckmann's text for prospective elementary teachers, since teaching and learning of this topic are similarly well-researched.

Analysis

For each task, we determined whether it explicitly applied mathematics to teaching. We defined a task as *explicitly applying mathematics to teaching* if it explicitly described any contextual feature of teaching. Such features were often signaled by phrases such as "suppose you are teaching..." or "a common student approach is..."; requests for a student-accessible explanation; embedded student work or statement of a teaching goal; or representations typically used at that level of teaching and not used beyond that level of teaching (e.g., algebra tiles for polynomials). We made no evaluation of the authenticity of the teaching situation. Table 2 provides two example tasks.

Table 2

Examples of tasks

Explicitly applies mathematics to teaching	*Does not explicitly apply mathematics to teaching*
"Three students are asked to produce an equation for the line passing through the points (1, 3) and (5, 9/2). The students each produce `different' final answers, namely, $y-3=\frac{3}{8}(x-1), y-\frac{3}{8}x=\frac{21}{8}$, and $8y-3x=21$. Are all of these equations correct? Discuss." (Bremigan, Bremigan, & Lorch (2011), Section 2.1.2, #4, p. 38)	"Prove that $Ax+By=C$ and $A'x+B'y=C'$ are equations for the same line if and only if there exists a nonzero real number λ such that $A'=\lambda A, B'=\lambda B$, and $C'=\lambda C$." (Bremigan, Bremigan, & Lorch (2011), Section 2.1.2, #6, p. 38)

Findings

We present two findings: one on the frequency of tasks that explicitly apply mathematics to teaching, and one on the nature of such tasks which may account for the contrast in frequency.

Frequency of Tasks

We found that among texts for prospective secondary teachers, there was a relatively low percentage of tasks that explicitly applied mathematics to teaching, as compared to those that did not. The percentages are summarized in Table 3.

We emphasize that we do not take the stance that tasks that do not explicitly apply mathematics to teaching are less worthwhile than tasks that do. However, given secondary teachers' perception of the irrelevance of content preparation, it behooves the field to examine the nature of tasks that do situate mathematics in teaching.

Table 3

Number of tasks that explicitly apply mathematics to teaching

Textbook	# Tasks	# Explicitly apply math to teaching
Usiskin et al. (2002)	198	3 (1.5%)
Bremigan, Bremigan, & Lorch (2011)	270	23 (8.5%)
Sultan & Artzt (2010)	129	28 (21.7%)
Beckmann (2011)	118	57 (48.3%)

Accounting for Contrast: "Variations on a theme" versus "one and done"

Consider the following three tasks from Beckmann (2011): "Discuss why it can be confusing to show an improper fraction such as 7/3 with pieces of pie or pieces of some other object. What is another way to show the fraction 7/3?"; "Erin says the tick mark [shown in a number line figure in the textbook] should be labeled 2.2. Is Erin right or not? If not, why not, and how can she label the tick mark properly"; "Liam says the tick mark [shown in a figure] should be labeled 1.7. Is Liam right or not? If not, why not, and how can the tickmark be labeled properly?" (p. 57).

These tasks typify opportunities in Beckmann (2011) to situate mathematics in teaching. The tasks ask the teacher to do specific work that arises in teaching: representing particular values on the number line and in other forms, as well as considering the limitations of common representations in the context of a particular example. These tasks are "variations on a theme," and it is easy to imagine adding other tasks in this theme that are authentic to teaching and that highlight distinguishing mathematical characteristics of different examples. In addition, explaining particular examples is an essential part of teaching practice (e.g., Leinhardt, 2001); thus a focus on specific examples may heighten these tasks' value as approximations of practice.

Consider now these two tasks from textbooks for the secondary level: "After doing the previous problem, one of your students asks if it is true that if we have a cubic polynomial with roots r, s, and t, then a polynomial that has roots $1/r$, $1/s$, and $1/t$ is just the polynomial with the coefficients reversed. How do you respond? Justify your answer" (Sultan & Artzt, 2011, p. 74); "... For 'ordinary functions' from $\mathbb{R}$ to $\mathbb{R}$ (the sort one studies in a calculus class), what are advantages of having data in tabular form? What are advantages of a graph? Try to think of several advantages for each" (Bremigan, Bremigan, & Lorch, 2010, p. 8).

These tasks contrast with those highlighted in Beckmann (2011) in that they fix a mathematical context, and then address this context in a way that is general in scope, rather than focusing attention on specific examples. These tasks, while not accounting for all tasks that explicitly apply mathematics to teaching in the secondary level textbooks analyzed, do account for many of them. Mathematical specificity/genericity is a salient trait because being either too generic or too specific gives less room for variation – and therefore fewer opportunities for deliberate practice (Ericsson, 2002). These tasks are "one and done."

Conclusion

We analyzed tasks from commonly used textbooks, and found that the nature and of tasks for the secondary level were more often "one and done" than "variations on a theme," and that the frequency of these tasks was low, ranging from 1.5 percent to 21 percent of possible tasks. By contrast, we found that a commonly used textbook for the elementary level featured a high frequency of tasks that applied mathematics to teaching, and that these tasks were "variations on a theme."

Tasks such as those in Beckmann (2011) present authentic mini-cases in which teachers can situate mathematics in teaching. In such tasks, teachers have the potential to develop their capacity for modeling

representations to students, to inquire how definitions determine particular representations, and through these variations, how explanations of individual specific fractions or values constitute a "generic" explanation that is powerful beyond the specific example in which it is situated (Mason & Pimm, 1984). "Variations on a theme" account for the high proportions of tasks that apply mathematics to teaching at the elementary level, and they also represent substantive opportunities for teachers to situate content knowledge in authentic cases of teaching.

In teaching, explanations often begin with or include examples that showcase important features of a concept (Leinhardt, 2001), and so overly generic tasks, while having the potential to apply to many situations, also obscure the decision making that may need to be made for particular examples. However, this observation also gives us hope for designing more and better tasks for prospective secondary teachers. One suggestion may be to take the more generic "one and done" problems, identify the work of teaching embedded in the task, and then make that work more contextual. Doing so may well increase the authenticity of tasks as approximations of teaching practice, as well as open opportunities for more deliberate practice.

For More Information

Contact Yvonne Lai, University of Nebraska-Lincoln (yvonnexlai@unl.edu).

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