The Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools[1]

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The Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools [MODULE(S^2)] Research Action Cluster (RAC) is focused on the development of prospective secondary mathematics teachers' (PSMTs') knowledge of mathematics content needed to support student learning. This focus addresses recommendations set forth in *The Mathematical Education of Teachers II* (Conference Board of the Mathematical Sciences [CBMS], 2012) for courses in secondary mathematics teacher preparation programs to provide opportunities for prospective teachers to "delve into the mathematics… while engaging in mathematical practice as described by the CCSS" (CBMS, 2012, p. 46). The work of the RAC aims to address the identified problem that undergraduate programs fail to lead teacher candidates to: a) deeply understand the mathematics they will actually teach and b) experience learning in a manner consistent with what will be expected of them as professional educators (Banilower et al., 2013).

In response to this problem, the MODULE(S^2) RAC has established the following objectives:

- Create twelve collaboratively designed modules aimed to develop PSMTs' mathematical knowledge for teaching algebra, geometry, modeling, and statistics in grades 6-12.
- Pilot and support the implementation of the modules.
- Revise the modules based on implementation data, instructor feedback, and PSMTs' work.
- Evaluate the effectiveness of modules with regards to their ability to develop PSMTs' mathematical knowledge for teaching.
- Disseminate the modules across multiple institutions, beginning with MTE-P institutions.

We adopted the Mathematical Knowledge for Teaching (MKT; Hill, Ball & Schilling, 2008) framework for our work. In this framework, subject matter knowledge for teaching

[1] The RAC Promo Sheet, presented during the opening of the conference to report on current activities of the RAC, can be found after the reference list.

mathematics includes the mathematics one teaches (Common Content Knowledge [CCK]), but it also includes knowing mathematics in a specialized way to meet the demands of teaching (Specialized Content Knowledge [SCK]) and the broader landscape of mathematics in which the mathematics one teaches is situated (Horizon Content Knowledge [HCK]). Pedagogical Content Knowledge (PCK) is also included in this framework, because it is specific to teaching mathematics. PCK includes three components in the MKT model: knowledge of how students conceive of particular content topics (Knowledge of Content and Students [KCS]), pedagogical principles for teaching specific content topics (Knowledge of Content and Teaching [KCT]), and knowledge of the curriculum resources available for the teaching of specific content and how to sequence their use to enhance student learning (Knowledge of Content and Curriculum [KCC]). Hill, Rowan and Ball (2005) showed that teachers' mathematical knowledge for teaching is positively correlated with student achievement, so growth of PSMTs' mathematical knowledge for teaching could have powerful effects on students' STEM achievement.

Current State of the Work

The proposed modules will be written with a common organizational structure. Materials for PSMTs will include activities, workbooks, excerpts from journals and secondary curricula, representations of practice, and homework sets. Materials for instructors will include instructor guides, assessment strategies, and teaching applications. Writing of modules is currently in various stages of production across the selected content areas.

In the 2014-2015 academic year, Emina Alibegovic and Alyson Lischka collaborated on writing and piloting three modules in geometry: (1) axiomatic systems, (2) rigid transformations, and (3) similarity and area. The modules include opportunities for PSMTs to analyze student work and depictions of classroom events through the Lesson*Sketch* platform (Herbst, Aaron, & Chieu, 2013). In the initial pilot year, participating PSMTs completed a pre- and post-test assessment of MKT (MKT-Geometry; Herbst & Kosko, 2012). Eighteen of the twenty-one students noted improvement in scores from pre-test to post-test. In Spring 2016, Alyson Lischka piloted revised materials and collected qualitative and quantitative data assessing the effectiveness of the materials in improving PSMTs' MKT. Pre- and post-test data was gathered using the Geometry Assessment for Secondary Teachers (Mohr-Schroeder, Ronau, Peters, Lee, & Bush, under review) to measure PSMTs' mathematical knowledge for teaching. The results indicated a statistically significant gain in geometric knowledge for teaching for the PSMTs. Preliminary analysis of qualitative data demonstrated a shift in PSMTs' ability to address specific mathematical ideas when responding to student thinking in a Lesson*Sketch* prompt. Further analysis is currently being conducted. Additional pilots are planned for the 2016-2017 school year.

The modeling writing team, Brynja Kohler and Cynthia Anhalt, has drafted three planned modules: (1) the modeling process, (2) classic models, and (3) in-depth modeling experience.

The project-based approach used in the modules, involving data gathering and creative model design activities, was inspired by labs that have been developed by Kohler and colleagues such as the Shrimp Diffusion Lab (Kohler, Swank, Haefner, & Powell, 2010), the Leaky Bucket Lab (Powell, Kohler, Haefner, & Bodily, 2012), and Coffee To Go (Kohler & Bruder, 2015).

Jason Aubrey and Yvonne Lai, have outlined three planned modules for Algebra on: (1) functions and relations, (2) complex numbers, and (3) ordered fields. An initial draft of the module on ordered fields focuses on properties of ordered fields which underlie the algebra curriculum in secondary mathematics.

In the 2014-2015 academic year, Andrew Ross and Stephanie Casey collaborated to write a statistics module addressing categorical association (including analysis of two-way tables, segmented bar graphs, and chi-squared randomization tests) and piloted it at 5 institutions. Analysis of the pre- and post-assessment data shows that the materials were effective at improving the participants' MKT for categorical association (Casey, Ross, Groth, & Zejnullahi, 2015; Casey, Zejnullahi, Wasserman, & Champion, 2015).

In addition to moving forward on writing and piloting modules, the MODULE(S^2) RAC has submitted an NSF proposal for an IUSE grant. Favorable comments were received from reviewers although the initial proposal was not funded. The team is currently working toward a resubmission of a revised proposal to fund the writing, analysis, and dissemination of this work. This project will not only develop materials that will increase PSMTs' MKT, it will also generate knowledge addressing important questions in the field of mathematics teacher education. The following questions are encompassed within the proposed future research assessing the effectiveness of the developed modules:

1. How do the MODULE(S^2) materials help PSMTs develop mathematical knowledge for teaching?
2. In what ways is the development of pre-service teachers' MKT related to their perception of content and skill at that content?
3. In what ways does pre-service teachers' MKT transfer between content context in which it was previously learned and new content contexts?
4. What are the necessary conditions to support implementation of the modules in current courses?

Moving Forward

Across the partners involved in the MODULE(S^2) RAC, reflections on existing piloted course materials have been positive. We have observed evidence that PSMTs increase their mathematical knowledge for teaching while learning mathematical content using the materials. We continue to revise and improve materials based on research findings. At many institutions, PSMTs complete College Geometry with the MODULE(S^2) materials prior to their Secondary

Mathematics Methods course. This is the case at Middle Tennessee State University, where it has been noted that mathematical knowledge for teaching that PSMTs gain in College Geometry serves as a vital foundation for discussions of practice in the methods course; PSMTs repeatedly point to the instruction in College Geometry as providing a connection to content they will teach while they concurrently engage in a deep exploration of geometric concepts.

Next steps include planned pilots of the geometry materials at additional MTE Partnership institutions and further work on materials in other content areas. This ongoing work will continue to contribute to the ways in which mathematics teacher educators can impact the mathematical knowledge for teaching developed among PSMTs at MTE partner institutions. Participation in the RAC's activities (including piloting) offers an opportunity for local partnerships to strengthen ties between members and stakeholders through the collaboration and local adaptation of the materials. We invite participating institutions to connect with the RAC and explore possible collaborations through writing, reviewing, and implementing the materials.

For More Information

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- Emina Alibegovic, Rowland Hall-St. Mark's School, eminaalibegovic@rowlandhall.org

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MTE-Partnership
Solicitation for Participation in the
MODULE(S)^2 RAC
April, 2016

Problem Addressed

During undergraduate study, prospective secondary mathematics teachers (PSMTs) do not always have the opportunity to build deep understanding of the mathematics they will be asked to teach. MODULE(S)^2 is concerned with developing mathematical knowledge and habits of mind for teaching for PSMTs.

We address *Candidates' Knowledge and Use of Mathematics* within the Teacher Candidate Knowledge, Skills and Dispositions section of the guiding principles as represented by the following indicators:

- Mathematical habits of mind
- Knowledge of the discipline
- Specialized knowledge of mathematics for teaching
- Nature of mathematics

General Approach

Our RAC aims to build communities among mathematicians, mathematics educators, and K-12 collaborators and work together to establish common content courses for mathematics teachers relevant to their professional needs. Because of the variety of needs and structures at different universities and teacher preparation programs, we have chosen to use a flexible format of creating course modules, each 3-5 weeks in length, that could be (a) used as a stand-alone mini course, (b) inserted into an existing course, or (c) combined to create one coherent course with the goal of creating a program in which all courses are carefully crafted to develop mathematical knowledge for teaching.

Our objectives are:

- Develop twelve collaboratively designed modules that focus on building a deep understanding of secondary mathematics themes identified in the CCSSM and the new CAEP accreditation standards.
- Investigate the impact of modules on PSMTs' opportunities to learn and readiness to teach school mathematics

Who We Are

- Geometry:
  - Emina Alibegovic, Department of Mathematics, University of Utah
  - Alyson Lischka, Department of Mathematical Sciences, Middle Tennessee State University
  - Taylor Haslam, Taylorsville High School, Granite District, Utah
- Modeling:
  - Brynja Kohler, Department of Mathematics and Statistics, Utah State University
  - Cynthia Anhalt, Department of Mathematics, University of Arizona
  - Brian Lawler, Mathematics Education, CSU San Marcos
- Statistics:
  - Stephanie Casey, Department of Mathematics, Eastern Michigan University
  - Andrew Ross, Department of Mathematics, Eastern Michigan University
  - Kady Schneiter, Department of Mathematics and Statistics, Utah State University
  - Joyce Smart, Logan High School, Logan City School District, Logan, Utah
- Algebra:
  - Jason Aubrey, Department of Mathematics, University of Arizona
  - Yvonne Lai, Department of Mathematics, University of Nebraska-Lincoln
- Master Editor:
  - Rachael Kenney, Department of Mathematics, Purdue University

Current Progress

- Geometry:
  - All modules completed
  - The course has been piloted at three institutions, two modules piloted separately at other institutions
  - In the process of editing and revising instructor and course materials
  - New pilots are scheduled for fall
- Modeling:
  - First module completed, first pilot scheduled for fall.
  - Modules 2 and 3 outlined, materials in development, ongoing research
- Statistics:
  - First module completed, and piloted at several institutions
  - Modules 2 and 3 outlined, materials in development, ongoing research
- Algebra:
  - First module completed, and piloted at several institutions
  - Modules 2 and 3 outlined, materials in development, ongoing research

Opportunities for Engagement

	Geometry	Modeling	Statistics	Algebra
Full partner				
Participating partner	Reviewing, editing, piloting*	Contributing, reviewing, editing, piloting*	Reviewing, editing, piloting*	Contributing, reviewing, editing, piloting*
Exploratory partner	Welcome	Welcome	Welcome	Welcome

* Partners involved with piloting will be asked to communicate frequently with the full partners. In addition, there will be data collection required for the purpose of course material improvement and revision.